

Measurements

Preparation for General Chemistry · Dr. Karmach

By the end of this unit, you can...

- Write and read numbers in scientific notation
- Report measurements and results with the correct significant figures
- Convert units with conversion factors, canceling units at each step
- Use density as a conversion factor between mass and volume
- Convert a temperature among $^{\circ}\text{C}$, $^{\circ}\text{F}$, and K, and handle a temperature difference

Today's route

1. **Scientific Notation**
2. **Significant Figures**
3. **The SI Unit System**
4. **Dimensional Analysis**
5. **Temperature Scales & Conversions**
6. **Density as a Conversion Factor**

1 · Scientific Notation

Write numbers in scientific notation and read them back (the decimal's move sets the exponent's size and sign, the coefficient stays at $1 \leq M < 10$), and enter them on a calculator with EE.

Numbers too big and too small to write out

Two numbers chemistry runs on

particles in one mole

602,214,076,000,000,000,000,000

$$= 6.022 \times 10^{23}$$

15 zeros just to hold the size

mass of one hydrogen atom

0.000000000000000000000000167 g

$$= 1.67 \times 10^{-24} \text{ g}$$

23 zeros just to hold the size

the coefficient keeps the digits · the exponent carries the size

Written out, chemistry's numbers are unreadable. Scientific notation writes each in a handful of characters: the digits once, then a power of ten carrying the size.

Powers of ten: one decimal place per step

$$10^3 = 1000 \cdot 10^2 = 100 \cdot 10^1 = 10 \cdot 10^0 = 1 \cdot 10^{-1} = 0.1 \cdot 10^{-2} = 0.01 \cdot 10^{-3} = 0.001$$

each step up multiplies by ten · each step down divides by ten

A positive exponent counts the zeros after the 1. A negative exponent means a number smaller than 1, never a negative number.

Write 10,000 and 0.00001 as powers of ten.

$$10,000 = 10^4 \cdot 0.00001 = 10^{-5}$$

a 1 followed by four zeros → 4 · the 1 sits five places right of the point → -5

One coefficient, one exponent

Scientific notation writes any number as a coefficient times a power of ten: the coefficient carries the digits, the exponent carries the size.

$$6.022 \times 10^{23}$$

coefficient M with $1 \leq M < 10$ · exponent n is a whole number · $10^{23} = \text{"times ten, 23 times"}$

Exactly one nonzero digit stands before the decimal point.

Standard → scientific: count the decimal's move

Slide the decimal until one nonzero digit leads. Each factor of ten moves it one place, so places moved = factors of ten = the exponent n . Direction sets the sign.

Moving the decimal point sets the exponent

how far it moves = the size of n · which way it moves = the sign of n

move 3 places right →

0 . 0 0 4 5 6



$$= 4.56 \times 10^{-3}$$

number < 1 → negative exponent

move 7 places left ←

9 3 0 0 0 0 0 0



$$= 9.3 \times 10^7$$

number > 1 → positive exponent

Write it as $M \times 10^n$ with $1 \leq M < 10$

moved RIGHT (was < 1) → $n < 0$

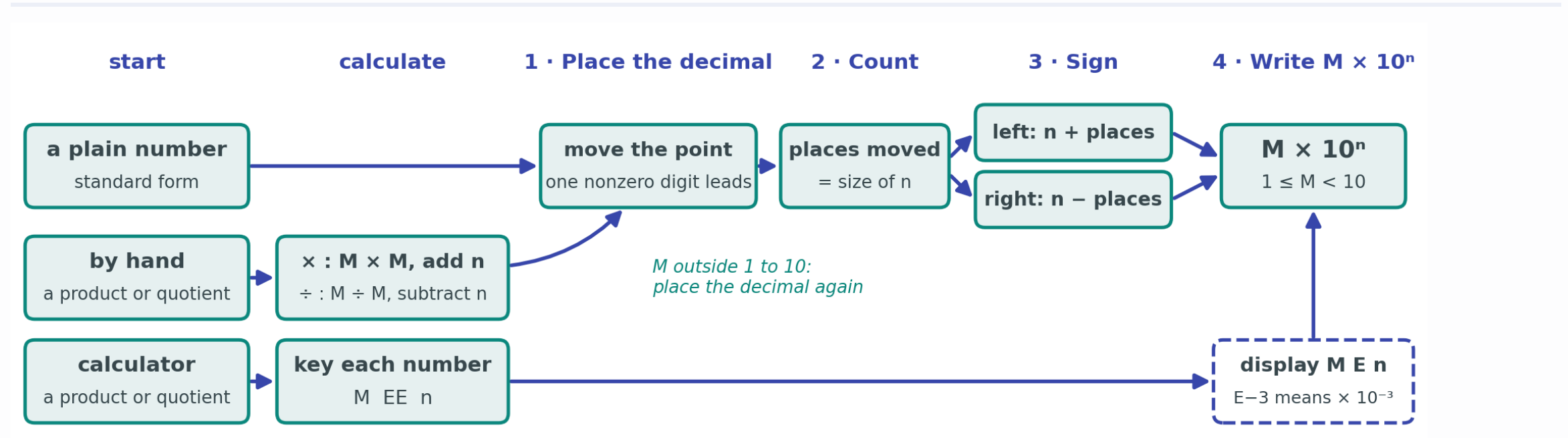
· moved LEFT (was > 1) → $n > 0$



The method: convert to scientific notation

1. Place the decimal so one nonzero digit leads: that is M .
2. Count the places moved: that is $|n|$.
3. Sign: left $\rightarrow$ add the places to n ; right $\rightarrow$ subtract them.
4. Write $M \times 10^n$.

One map for every conversion



A plain number runs Steps 1 to 4. A calculator display already reads $M \times 10^n$. A coefficient worked out by hand that lands outside 1 to 10 goes back through Steps 1 to 3.



Guided example: the radius of a gold atom

0.000000000166 m

given: radius of one gold atom · wanted: the same length as $M \times 10^n$ m

A gold atom's radius is about **0.000000000166 m**. Written out, the zeros hide the size.

Write the radius in scientific notation.

✓ Guided example: solution

0.000000000166 m

given: radius of one gold atom · wanted: $M \times 10^n$ m

Step 1 · Place the decimal so one nonzero digit leads

Step 2 · Count the places moved

0.000000000166 $\rightarrow$ 1.66 · 9 zeros + 1 digit = **10 places**

Step 3 · Sign

Step 4 · Write $M \times 10^n$

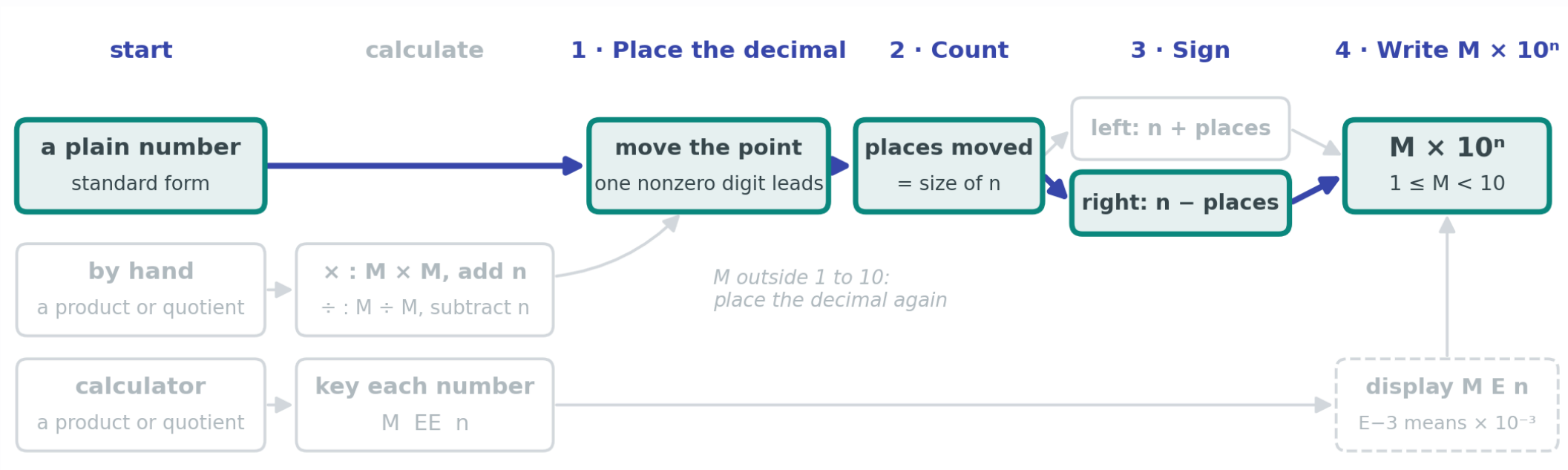
0.000000000166 m = **1.66×10^{-10} m**

 **Sense check:** An atom is far smaller than 1 m, so a large negative exponent fits. All three digits, 1, 6 and 6, stay in M. ✓

✓ Guided example: the route on the map

$$0.000000000166 \text{ m} = 1.66 \times 10^{-10} \text{ m}$$

given: radius of one gold atom · found: $M = 1.66 \cdot n = -10$



🧠 **Sense check:** A plain number below 1: the decimal moves right, so the places come off n. ✓



Practice 1

Earth to Sun: 149,600,000 km

given: 149,600,000 km · wanted: n in $M \times 10^n$ km

The Sun sits **149,600,000 km** from Earth. Written in scientific notation, what is the exponent n ?

- A. -8
- B. 8
- C. 5
- D. 7

✓ Practice 1: answer B

$$149,600,000 \text{ km} = 1.496 \times 10^8 \text{ km}$$

$$M = 1.496 \cdot \text{decimal moves 8 places left} \cdot \text{number} > 1 \rightarrow n = +8$$

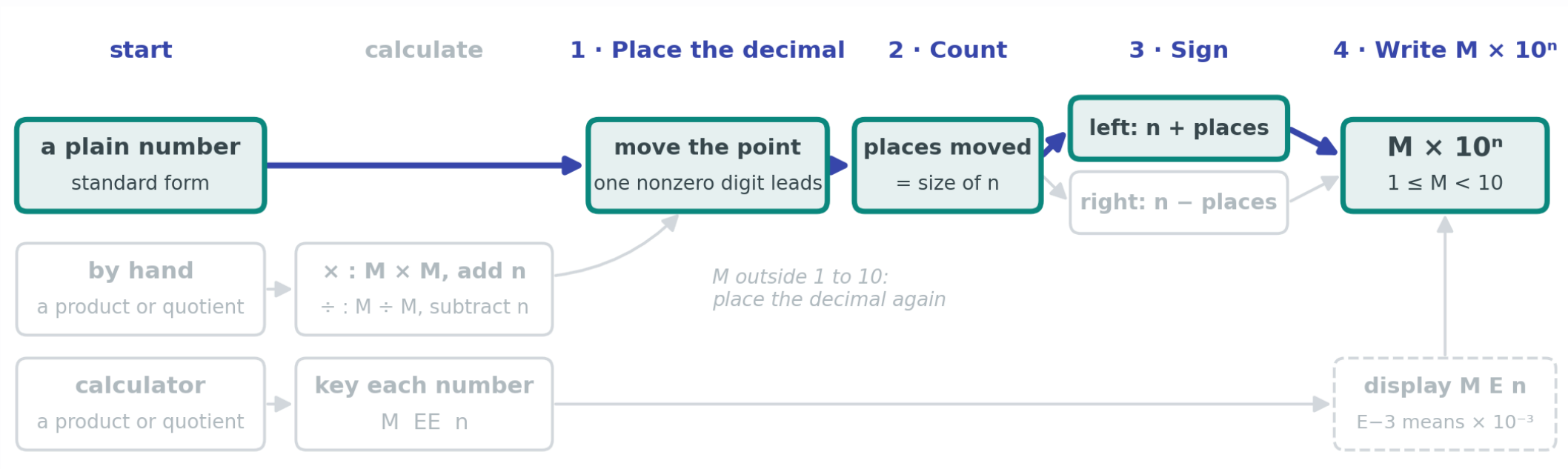
B: the point moves left past 4, 9, 6 and five zeros, 8 places. A flipped the sign: a number above 1 takes a positive n . C counted only the five zeros and skipped 4, 9 and 6. D stopped one place early: 14.96×10^7 leaves two digits before the point.

 **Sense check:** 10^8 is 100,000,000, and 149,600,000 is about 1.5 of those. ✓

✓ Practice 1: the route on the map

$$149,600,000 \text{ km} = 1.496 \times 10^8 \text{ km}$$

given: 149,600,000 km · found: $n = 8$



🧠 **Sense check:** A plain number above 1: the decimal moves left, so the places add to n. ✓



Worked example 1: two conversions

0.00456 and 93,000,000

write each in scientific notation, $M \times 10^n$

One is small (less than 1), one is large (greater than 1). Count the decimal's move for each, then read the sign from the direction.

✓ Worked example 1: solution

The small number moves right (negative n); the large number moves left (positive n).

$$0.00456 = 4.56 \times 10^{-3}$$

$$93,000,000 = 9.3 \times 10^7$$

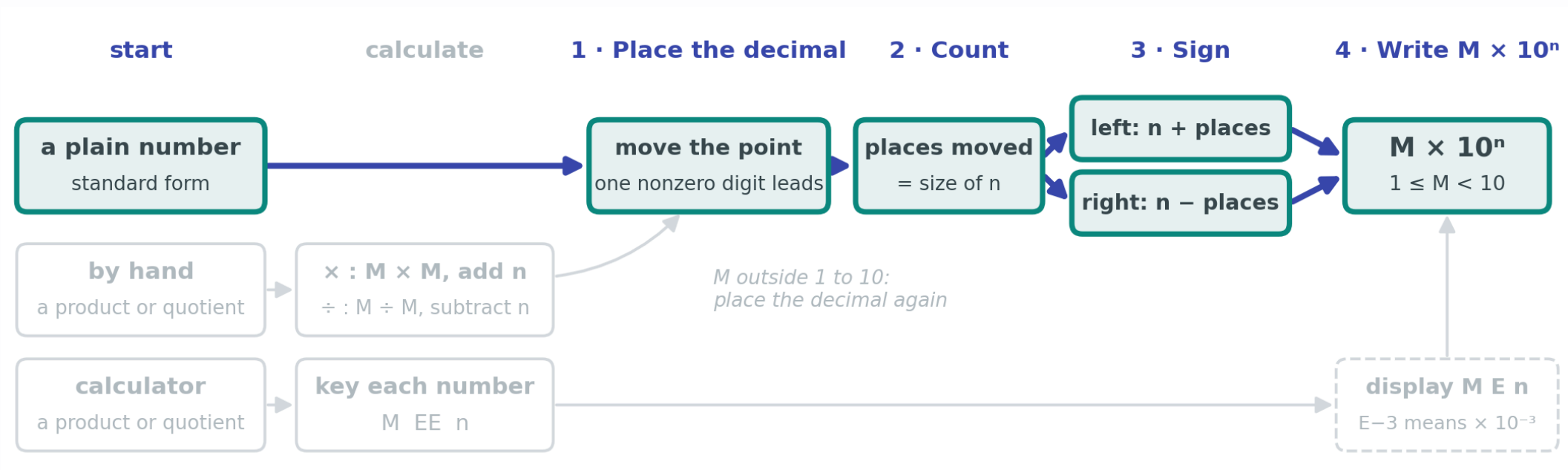
3 places right $\rightarrow n = -3$ · 7 places left $\rightarrow n = +7$

🧠 **Sense check:** A leading zero forces n negative; a long tail of zeros forces n positive.

✓ Worked example 1: the route on the map

$$0.00456 = 4.56 \times 10^{-3} \cdot 93,000,000 = 9.3 \times 10^7$$

given: two plain numbers · found: $n = -3$ and $n = +7$



🧠 **Sense check:** Same steps for both numbers. Only Step 3 differs: right for the small one, left for the large one. ✓

Scientific → standard: move the decimal back

Reverse it: the exponent says how many places to slide the decimal, and its sign says which way. Positive → right (bigger); negative → left (smaller).

$$9.3 \times 10^7 \rightarrow 93,000,000 \cdot 4.56 \times 10^{-3} \rightarrow 0.00456$$

+7: shift right 7 places, filling zeros · -3: shift left 3 places

 Your turn: convert 0.00072

Fill each blank, then check.

step	value
coefficient M	_____
decimal moves	_____
exponent n	_____

$$0.00072 = 7.2 \times 10^{-4}$$

move right 4 places · number < 1 $\rightarrow n = -4 \cdot M = 7.2$



Worked example 2: entering 6.022×10^{23}

6.022×10^{23} on a calculator

given: 6.022×10^{23} · wanted: the keying that enters it as one number

Every mole calculation starts by keying this number in. The EE key (EXP on some models) means "times ten to the".

A common first attempt keys the $\times 10$ out loud: 6.022×10 EE 23. Test it.

✓ Worked example 2: solution

6.022×10^{23} on a calculator

EE means "times ten to the" · wanted: one number in the machine

A common first attempt

keyed 6.022×10 EE 23 → the machine holds $6.022 \times 10 \times 10^{23} = 6.022 \times 10^{24} \times$
EE already supplies the $\times 10$ · keying $\times 10$ again multiplies in an extra ten

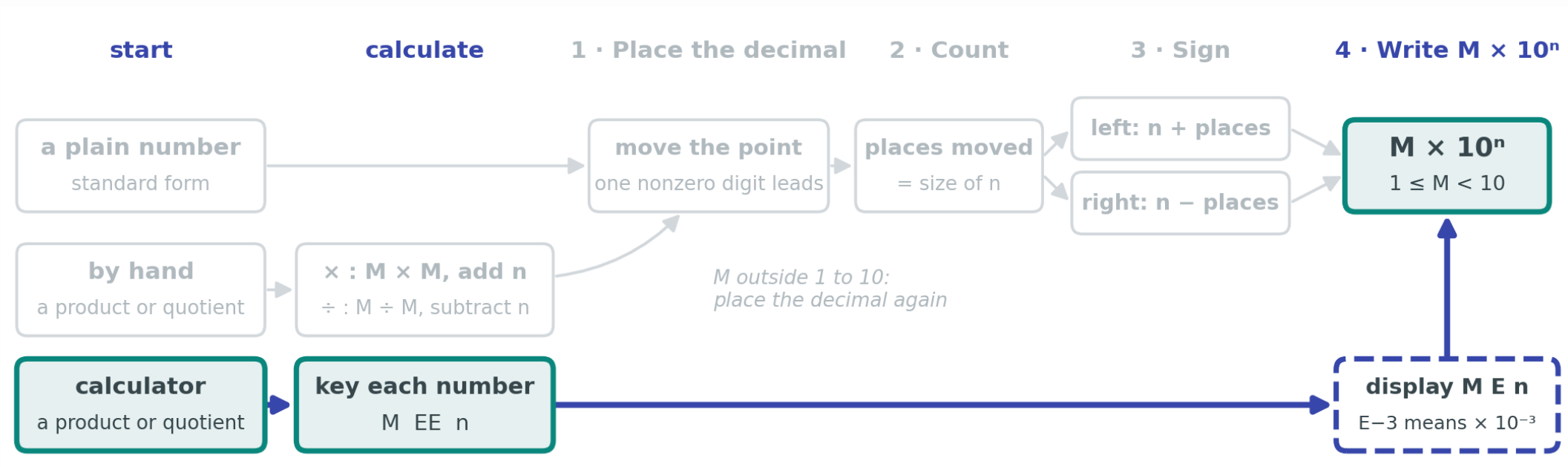
The correct keying

keyed 6.022 EE 23 → display $6.022\text{E}23 = 6.022 \times 10^{23}$ ✓
the whole number enters as one object: coefficient, then EE, then exponent

 **Sense check:** Read the display back before computing: E23 on the screen is $\times 10^{23}$. A result ten times too large marks the $\times 10$ -before-EE slip.

✓ Worked example 2: the route on the map

keyed 6.022 EE 23 → display 6.022E23 = 6.022×10^{23}
 given: 6.022×10^{23} · found: the keying 6.022 EE 23



🧠 **Sense check:** EE enters M and n together. Steps 1 to 3 never run: the display already reads $M \times 10^n$. ✓

⚠ Where this goes wrong

Sign backwards. A number below 1 gets a negative n: $0.00456 = 4.56 \times 10^{-3}$, not 10^3 .

The $\times 10$ -before-EE slip. On a calculator, 6.022 EE 23 is 6.022×10^{23} ; keying 6.022×10 EE 23 multiplies in an extra ten.

Off-by-one count. Count decimal moves, not zeros: 93,000,000 moves 7 places $\rightarrow 10^7$.



Practice 2

width of one human hair: 0.0000706 m

given: 0.0000706 m · wanted: the width as $M \times 10^n$ m

A human hair is **0.0000706 m** wide. What is that width in meters, written in scientific notation?

- A. 7.6×10^{-5}
- B. 7.06×10^5
- C. 7.06×10^{-4}
- D. 7.06×10^{-5}

✓ Practice 2: answer D

$$0.0000706 \text{ m} = 7.06 \times 10^{-5} \text{ m}$$

$$M = 7.06 \cdot \text{decimal moves 5 places right} \cdot \text{number} < 1 \rightarrow n = -5$$

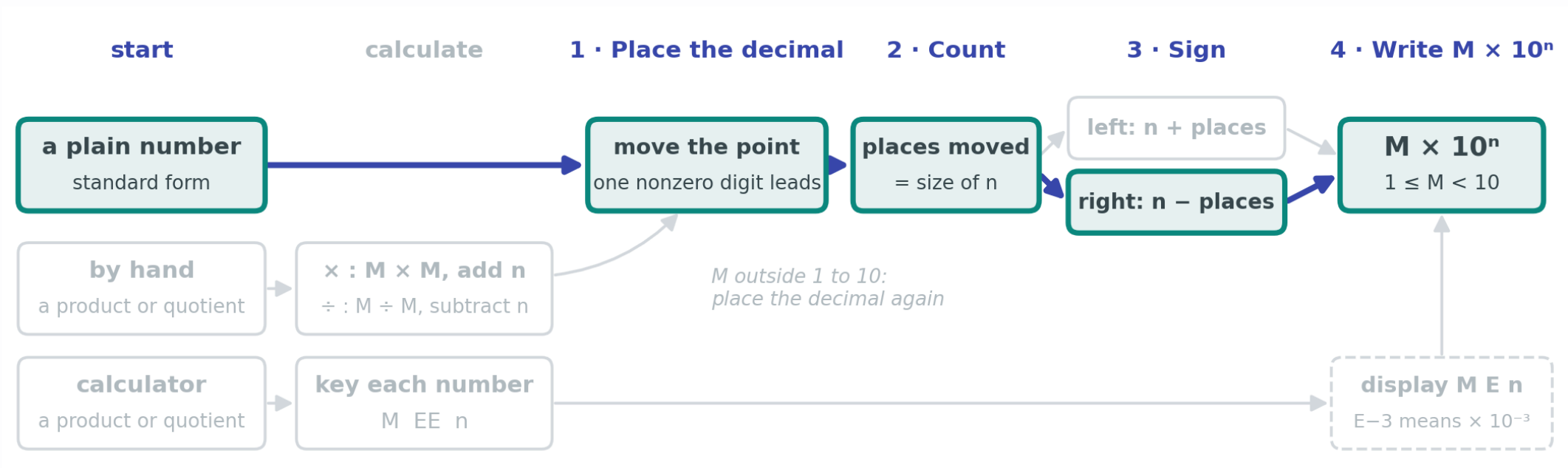
D: the point moves right past four zeros and the 7, 5 places, and M keeps all three digits. A dropped the zero between 7 and 6: a zero caught between nonzero digits is a digit of M. B flipped the sign: a number below 1 takes a negative n. C counted the four zeros and skipped the 7.

 **Sense check:** Back to standard: -5 slides the point 5 places left, $7.06 \rightarrow 0.0000706$. ✓

✓ Practice 2: the route on the map

$$0.0000706 \text{ m} = 7.06 \times 10^{-5} \text{ m}$$

given: $0.0000706 \text{ m} \cdot \text{found: } M = 7.06 \cdot n = -5$



🧠 **Sense check:** A plain number below 1: the decimal moves right, so the places come off n. ✓

 Practice 3

one grain of fine sand: $2.0 \times 10^{-6} \text{ g}$

a pinch: 4.5×10^3 grains

A pinch of fine sand holds 4.5×10^3 grains, each of mass $2.0 \times 10^{-6} \text{ g}$. What is the mass of the pinch, in grams?

- A. 9.0×10^9
- B. 9.0×10^{-2}
- C. 9.0×10^{-3}
- D. 9.0×10^3

✓ Practice 3 · answer: C

keyed 2.0 EE -6 × 4.5 EE 3 → display 9.0E-3 = 9.0×10^{-3} g (answer C)

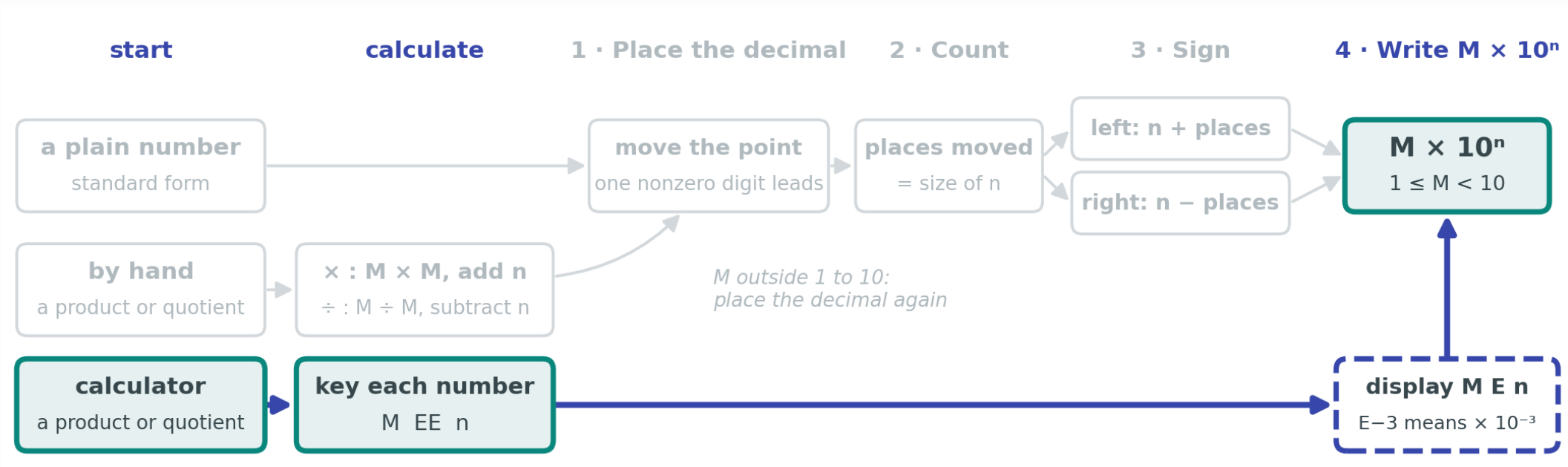
each number enters as one object · E-3 reads back as $\times 10^{-3}$

A keyed the -6 as +6: $2.0 \times 10^6 \times 4.5 \times 10^3 = 9.0 \times 10^9$. B keyed $\times 10$ before EE, an extra ten: $2.0 \times 10 \times 10^{-6} \times 4.5 \times 10^3 = 9.0 \times 10^{-2}$. D read the display back with the sign dropped: E-3 is 10^{-3} , not 10^3 .

🧠 **Sense check:** A pinch of sand is a few milligrams: 9.0×10^{-3} g = 0.0090 g. ✓

✓ Practice 3: the route on the map

keyed $2.0 \text{ EE } -6 \times 4.5 \text{ EE } 3 \rightarrow \text{display } 9.0\text{E}-3 = 9.0 \times 10^{-3} \text{ g}$
 given: $2.0 \times 10^{-6} \text{ g per grain} \cdot 4.5 \times 10^3 \text{ grains} \cdot \text{found: } 9.0 \times 10^{-3} \text{ g}$



🧠 **Sense check:** Each number went in with EE, and the display read straight into Step 4. ✓

Multiplying and dividing by hand

$$10^4 \times 10^{-7} = 10^{4+(-7)} = 10^{-3} \cdot 10^4 \div 10^{-7} = 10^{4-(-7)} = 10^{11}$$

multiply: add the exponents · divide: subtract the exponents · subtracting a negative adds

Coefficients multiply or divide; exponents add or subtract. A coefficient outside 1 to 10 gets its decimal placed again: left adds the places to n, right subtracts them. Plain numbers start at $n = 0$.



Worked example 3: a product by hand

$$(6.0 \times 10^3)(5.0 \times 10^{-5})$$

given: 6.0×10^3 and 5.0×10^{-5} · wanted: the product as $M \times 10^n$

Multiply the coefficients and add the exponents. Then write the product in scientific notation.

✓ Worked example 3: solution

$$(6.0 \times 10^3)(5.0 \times 10^{-5})$$

given: 6.0×10^3 and 5.0×10^{-5} · wanted: $M \times 10^n$

Multiply · coefficients times, exponents added

$$(6.0 \times 5.0) \times 10^{3+(-5)} = 30. \times 10^{-2}$$

Step 1 · Place the decimal so one nonzero digit leads

Step 2 · Count the places moved

Step 3 · Sign

Step 4 · Write $M \times 10^n$

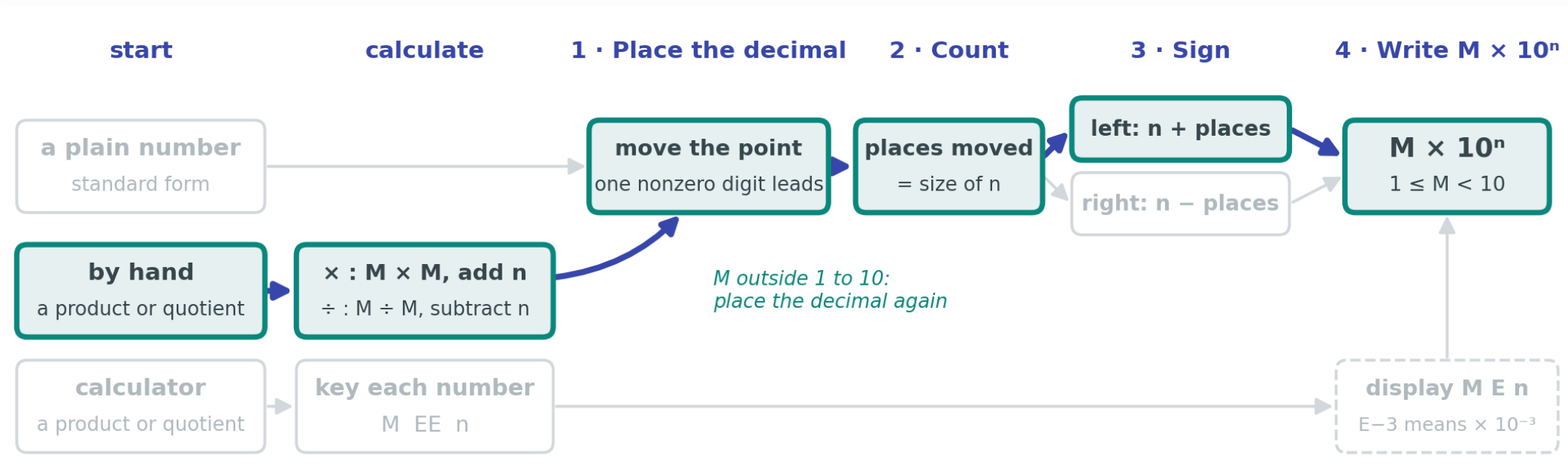
$$30. \times 10^{-2} = 3.0 \times 10^{-1}$$

 **Sense check:** A calculator keyed 6.0 EE 3 × 5.0 EE –5 shows 0.3, which is 3.0×10^{-1} . ✓

✓ Worked example 3: the route on the map

$$(6.0 \times 10^3)(5.0 \times 10^{-5}) = 30. \times 10^{-2} = 3.0 \times 10^{-1}$$

given: 6.0×10^3 and 5.0×10^{-5} · found: 3.0×10^{-1}



🧠 Sense check: The by-hand coefficient, 30., sat outside 1 to 10, so it went back through Steps 1 to 3. Moved left: n goes up by 1. ✓



Practice 4

6.0×10^{-7} g pollen per m^3 · 1.5×10^{-9} g per grain · 4.0×10^2 m^3 of air
given: pollen mass per cubic meter · mass of one grain · air volume · wanted: number of grains

On a high-pollen day, each cubic meter of outdoor air carries 6.0×10^{-7} g of pollen. One grain has a mass of 1.5×10^{-9} g. How many pollen grains are in 4.0×10^2 m^3 of that air?

- A. 1.6×10^5
- B. 1.6×10^4
- C. 6.3×10^{-6}
- D. 1.6×10^3
- E. 1.6×10^{-13}

✓ Practice 4: answer A

$6.0 \times 10^{-7} \text{ g pollen per m}^3 \cdot 1.5 \times 10^{-9} \text{ g per grain} \cdot 4.0 \times 10^2 \text{ m}^3 \text{ of air}$
two moves: pollen mass in the air, then grains in that mass

$$4.0 \times 10^2 \text{ m}^3 \times 6.0 \times 10^{-7} \text{ g/m}^3 = 24 \times 10^{-5} \text{ g} = \mathbf{2.4 \times 10^{-4} \text{ g}}$$

$$2.4 \times 10^{-4} \text{ g} \div 1.5 \times 10^{-9} \text{ g per grain} = 1.6 \times 10^{-4-(-9)} = \mathbf{1.6 \times 10^5 \text{ grains (answer A)}}$$

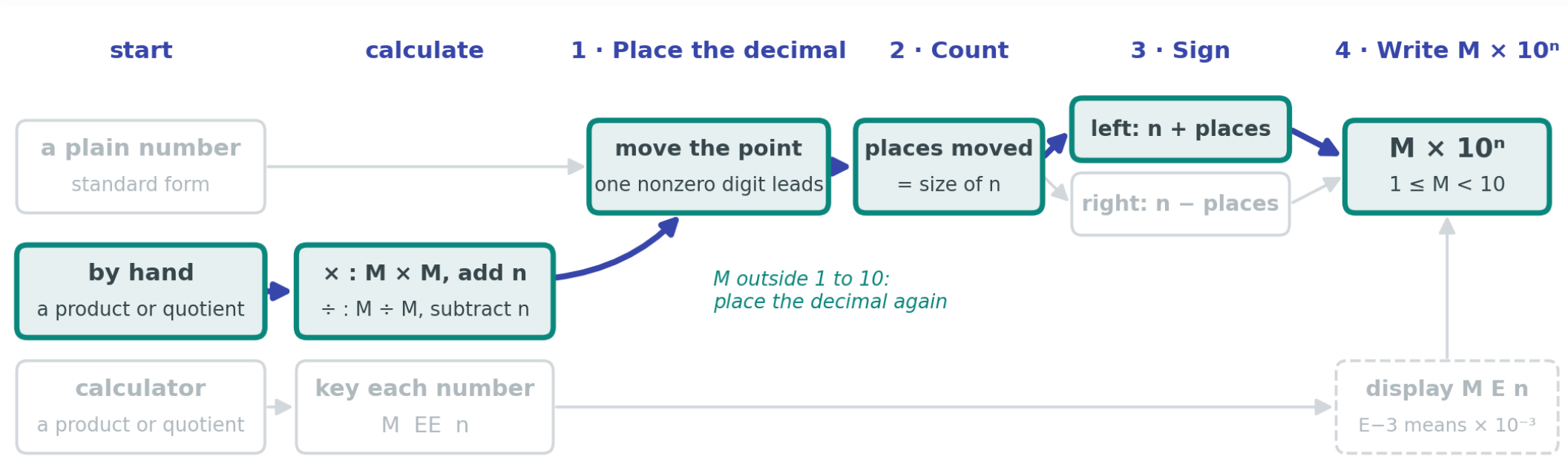
B moved the decimal in 24 but left n at -5 : $2.4 \times 10^{-5} \text{ g}$ gives 1.6×10^4 . Keying $1.5 \times 10 \text{ EE } -9$ lands there too. C divided the grain's mass by the pollen: $1.5 \times 10^{-9} \div 2.4 \times 10^{-4} = 6.3 \times 10^{-6}$. D moved n the wrong way: 24×10^{-5} became $2.4 \times 10^{-6} \text{ g}$, which gives 1.6×10^3 . E subtracted 9 instead of -9 : $10^{-4-9} = 10^{-13}$.

🧠 **Sense check:** Each cubic meter holds $6.0 \times 10^{-7} \div 1.5 \times 10^{-9} = 400$ grains, and $400 \text{ m}^3 \times 400 = 1.6 \times 10^5$. ✓

✓ Practice 4: the route on the map

$$6.0 \times 10^{-7} \text{ g/m}^3 \times 4.0 \times 10^2 \text{ m}^3 = 24 \times 10^{-5} \text{ g} = 2.4 \times 10^{-4} \text{ g}$$

given: $6.0 \times 10^{-7} \text{ g per m}^3 \cdot 1.5 \times 10^{-9} \text{ g per grain} \cdot 4.0 \times 10^2 \text{ m}^3 \cdot \text{found: } 1.6 \times 10^5 \text{ grains}$

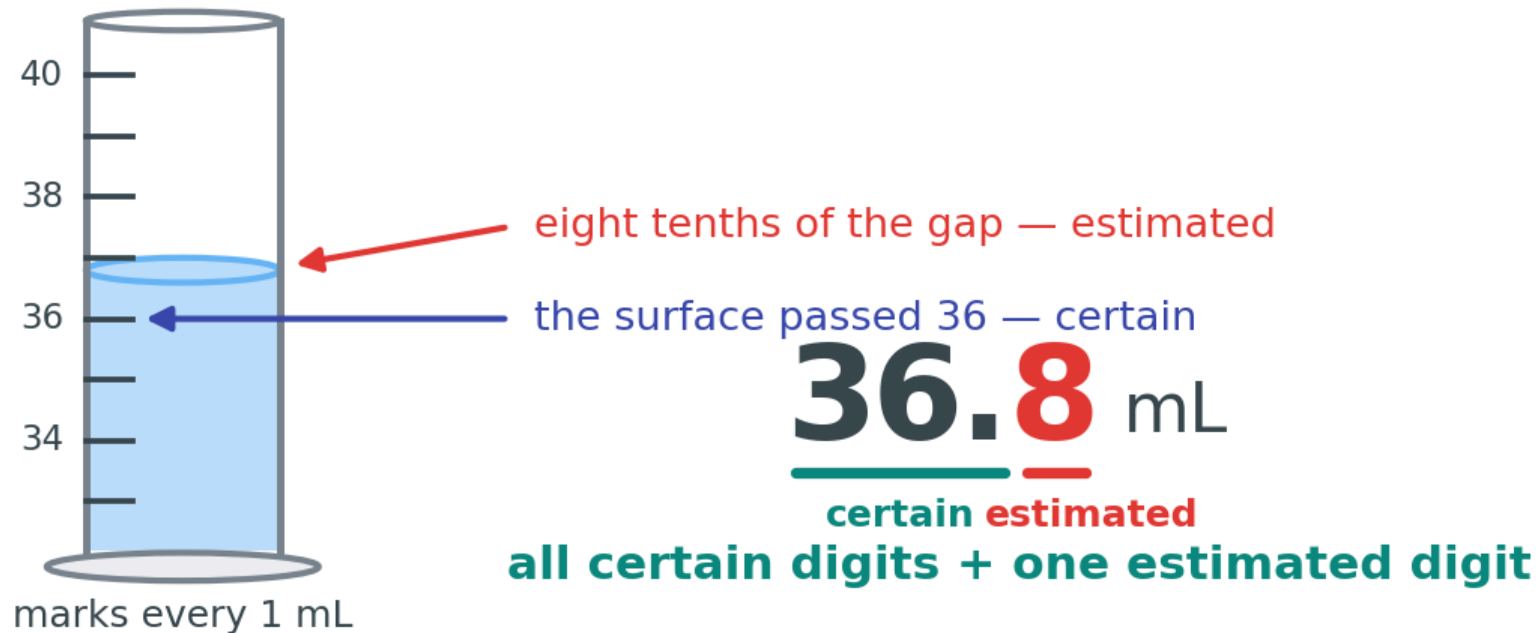


Sense check: The product 24×10^{-5} went back through Steps 1 to 3. The division needed no repair: 1.6 already lies between 1 and 10. ✓

2 · Significant Figures

Count the significant figures in any measurement, and round a calculated result with the rule that matches the operation.

Reading an instrument



The cylinder is marked every 1 mL. The water sits most of the way from 36 to 37: record 36.8 mL. The 3 and 6 are certain; the 8 is estimated.

Every measurement has three parts

36.8 mL

number: 36.8 · unit: mL · uncertainty: the last digit, 8, is estimated

A measurement reports a number, a unit, and how certain the number is. Two students read the same cylinder: 36.7 mL and 36.9 mL. Which digits agree?

36.7 mL · 36.9 mL

3 and 6 agree: certain · the last digit differs: estimated

Every reading ends in one estimated digit.

A measurement's digits record its certainty

36.8 mL

3, 6 certain · 8 estimated: the true volume lies between 36.7 and 36.9 mL

An instrument reports every digit it can distinguish, plus one estimated digit. Those digits are the **significant figures**. Writing 36.8 mL states the volume is known to the tenths and no further.

A better instrument gives more digits

36.8 mL (graduated cylinder)
estimated in the tenths → 3 sig figs

36.82 mL (burette)
estimated in the hundredths → 4 sig figs

A burette marks every 0.1 mL, so its estimate lands in the hundredths. Recording 36.82 mL from a cylinder marked in whole milliliters claims a digit that was never measured.

Exact numbers

$$24 \text{ tablets} \cdot 1 \text{ kg} = 1000 \text{ g}$$

counted · defined → no estimated digit → exact, unlimited sig figs

24.31 g

measured: the final 1 is estimated → 4 sig figs

Counted objects and defined relationships are not measurements. Nothing in them is estimated, so they have unlimited significant figures. Only measured values limit a result.



The method

1. **Nonzero digits** always count.
2. **Captive zeros** (between nonzero digits) always count.
3. **Leading zeros** (before the first nonzero digit) never count.
4. **Trailing zeros** count only with a decimal point.

One map for every sig-fig question

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
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REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or − → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

Counting tests each digit of one number. A calculated result takes its rule from the operation, then rounds once. Exact numbers never limit either.



Guided example: a burette reading

20.50 mL

read from a burette marked every 0.1 mL

A burette delivers acid into a flask. Count the significant figures in the reading. Run the four rules in order.

✓ Guided example: solution

20.50 mL

read from a burette marked every 0.1 mL

Step 1 • Nonzero digits always count

Step 2 • Captive zeros always count

Step 3 • Leading zeros never count

Step 4 • Trailing zeros count only with a decimal point

20.50 mL → 4 sig figs

counted: 2, 0, 5, 0 • no placeholders

 **Sense check:** The burette is marked every 0.1 mL, so its estimate lands in the hundredths: the final 0 is the estimated digit. Writing 20.5 mL would throw a measured digit away. ✓

✓ Guided example: the route on the map

20.50 mL

found: 4 sig figs · counted: 2, 0, 5, 0

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
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REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

 **Sense check:** Three digit tests did the work: nonzero, captive, and trailing with a decimal point. One number, no calculation: the bottom row stays unlit. ✓



Practice 1

100.50 g

a beaker of water on a top-loading balance

How many significant figures are in the balance reading?

- A. 2
- B. 3
- C. 4
- D. 5

✓ Practice 1 · answer: D

100.50 g → 5 sig figs (answer D)

nonzero: 1, 5 · captive: the two zeros of 100 · trailing with a decimal point: the final 0

A counted only the nonzero digits: 2. B read the zeros of 100 as placeholders: 3. They sit between the 1 and the 5, so they are captive. C dropped the trailing zero after the decimal point: 4.

 **Sense check:** The balance reads to the hundredths, so the final 0 is its estimated digit. All five digits were measured. ✓

✓ Practice 1: the route on the map

100.50 g

found: 5 sig figs · counted: 1, 0, 0, 5, 0

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit $4.3 \cdot 27$ counts	captive zero $403 \cdot 2.05$ counts	leading zero $0.04 \cdot 0.0072$ never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
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REPORT a calculated result



 **Sense check:** The path of the guided example: nonzero, captive, trailing with a decimal point. No leading zero and no placeholder. ✓



Worked example 1: a mass from the balance

0.04030 g

read from an analytical balance

An analytical balance reports the mass of a powder sample. Count the significant figures: test each digit against the four rules.

✓ Worked example 1: solution

0.04030 g

read from an analytical balance

Step 1 · Nonzero digits always count

Step 2 · Captive zeros always count

Step 3 · Leading zeros never count

Step 4 · Trailing zeros count only with a decimal point

0.04030 g → 4 sig figs

counted: 4, 0, 3, 0 · not counted: the two leading zeros

🧠 **Sense check:** In scientific notation the placeholders vanish: 4.030×10^{-2} g shows exactly the four significant digits.

✓ Worked example 1: the route on the map

0.04030 g

found: 4 sig figs · counted: 4, 0, 3, 0 · not counted: the two leading zeros

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result



🧠 **Sense check:** All four digit rules ran on one number. Only the leading zeros were left out. ✓



Worked example 2: a scale with no decimal point

1200 kg

a truck-scale reading, no decimal point

A truck scale reports the mass of a loaded pallet. A common first attempt: four written digits, four significant figures. Test it against the rules.

✓ Worked example 2: solution

1200 kg

a truck-scale reading, no decimal point

Step 1 • Nonzero digits always count

Step 2 • Captive zeros always count

Step 3 • Leading zeros never count

Step 4 • Trailing zeros count only with a decimal point

1200 kg → 2 sig figs

counted: 1, 2 • placeholders: 0, 0 • the four-digit count fails Step 4

1200. kg → 4 sig figs • 1.20×10^3 kg → 3 sig figs

the decimal point makes trailing zeros count • the coefficient shows only significant digits

🧠 **Sense check:** Two sig figs: the thousands digit is certain, the hundreds digit is the estimate. Nothing in 1200 records the tens or ones.

✓ Worked example 2: the route on the map

1200 kg

found: 2 sig figs · counted: 1, 2 · placeholders: 0, 0

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result



🧠 **Sense check:** With no decimal point, the trailing zeros take the placeholder branch. Only the nonzero digits count.



Take-home: trailing zeros need a decimal point

the same reading, three ways to write it

1200 kg

2 sig figs

no decimal point:
the zeros only hold place

1200. kg

4 sig figs

the decimal point makes
every trailing zero count

1.20 × **10³** kg

3 sig figs

the coefficient shows
only significant digits

■ significant ■ placeholder

1200 kg, 1200. kg, and 1.20×10^3 kg describe the same load with different certainty. To mark a trailing zero significant, write the decimal point or use scientific notation.

Your turn: count the sig figs

measurement	sig figs
0.0250 L	_____
305.0 g	_____
8700 m	_____

The decimal point decides every trailing zero.

0.0250 L → 3 · 305.0 g → 4 · 8700 m → 2

trailing zero after a decimal point counts · captive and trailing count · no decimal point: placeholders

⚠ Where this goes wrong

Counting trailing zeros with no decimal point. 2600 kg reads as four sig figs. Step 4 gives two: the zeros only hold place. Written 2600. kg, it has four.

Counting the leading zeros. In 0.0250 L, starting the count at the zeros gives 4 or 5. Leading zeros only locate the decimal point: 3 sig figs.

Dropping the trailing zero after a decimal point. The final zero of 0.0250 L was measured, and the decimal point makes it count: 3 sig figs, not 2.

Counting every written digit. 0.0250 L shows five digits but 3 sig figs. A digit is significant when it was measured, not when it is written.

 Practice 2

0.02060 g

the mass of a grain of rice on an analytical balance

How many significant figures does the measurement carry?

- A. 3
- B. 4
- C. 5
- D. 6

✓ Practice 2 · answer: B

0.02060 g → 4 sig figs (answer B)

counted: 2, 0, 6, 0 · not counted: the two leading zeros

The 2 and 6 count, the captive zero between them counts, and the decimal point makes the final zero count. A dropped the trailing zero: 3. C started counting at the first zero after the decimal point: 5. D counted every written digit: 6.

 **Sense check:** Scientific notation strips the placeholders: 2.060×10^{-2} g keeps exactly four digits.

✓ Practice 2: the route on the map

0.02060 g

found: 4 sig figs · counted: 2, 0, 6, 0 · not counted: the two leading zeros

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result



🧠 **Sense check:** The path of worked example 1: all four digit rules, with only the leading zeros left out. ✓

Rounding off

$$7.8342 \rightarrow 7.83$$

first dropped digit 4: below 5, the kept digit stays

$$0.4267 \rightarrow 0.43$$

first dropped digit 6: 5 or more, the kept digit rounds up

A calculator returns more digits than a measurement supports. Keep the significant ones and look at the first digit dropped: below 5, keep; 5 or more, round up.

Multiplication and division: fewest sig figs

$$4.20 \times 1.1 = 4.62 \rightarrow 4.6$$

3 sig figs $\times$ 2 sig figs $\rightarrow$ report 2 sig figs

A result can be no more certain than its least certain measurement. For multiplication and division, the answer keeps the **fewest sig figs** found among the inputs.



Worked example 3: volume of a block

$$8.5 \text{ cm} \times 4.27 \text{ cm} \times 1.36 \text{ cm}$$

given: three measured edges · wanted: the volume, correctly reported

A metal block's three edges are measured. Compute the volume and report it with the correct number of sig figs.

✓ Worked example 3: solution

$$8.5 \text{ cm} \times 4.27 \text{ cm} \times 1.36 \text{ cm}$$

sig figs: 2 · 3 · 3

Count each factor's sig figs

Multiply, then round to the fewest

$$8.5 \times 4.27 \times 1.36 = 49.3612 \text{ cm}^3 \text{ (calculator)} \rightarrow 49 \text{ cm}^3$$

reported to 2 sig figs, set by the 8.5

 **Sense check:** The 8.5 cm edge was estimated in the tenths. Two sig figs is everything those rulers measured; the calculator's extra digits were never measured at all.

✓ Worked example 3: the route on the map

$$8.5 \text{ cm} \times 4.27 \text{ cm} \times 1.36 \text{ cm} = 49 \text{ cm}^3$$

found: 49 cm^3 · 2 sig figs, set by the 8.5

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

🧠 **Sense check:** Count each factor, keep the fewest sig figs, round once: $49.3612 \rightarrow 49 \text{ cm}^3$. The + or – rule stays unlit. ✓



Practice 3

50 paper clips · 43.18 g

given: 50 identical clips in a box · 43.18 g on a balance · wanted: g per clip

A box of **50** identical paper clips has a mass of **43.18 g**. What is the mass of one paper clip, in grams?

- A. 0.8636
- B. 0.9
- C. 2159
- D. 1.158

✓ Practice 3 · answer: A

$$43.18 \text{ g} \div 50 = 0.8636 \text{ g per clip (answer A)}$$

43.18 g: measured, 4 sig figs · 50 clips: counted, exact · fewest among the measured values: 4

B read the count as a measurement with one sig fig: $0.8636 \rightarrow 0.9$. C multiplied: $43.18 \times 50 = 2159$, heavier than the whole box. D flipped the division: $50 \div 43.18 = 1.158$ clips per gram.

 **Sense check:** Counted objects carry no estimated digit, so the 50 never limits. The balance's four sig figs set the answer. ✓

✓ Practice 3: the route on the map

$$43.18 \text{ g} \div 50 = 0.8636 \text{ g}$$

found: 0.8636 g per clip · 4 sig figs, set by the balance

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

🧠 **Sense check:** The exact count skips the digit tests. Only 43.18 g is counted, and × or ÷ keeps its four sig figs. ✓



Worked example 4: adding two volumes

$$108.2 \text{ mL} + 1.66 \text{ mL}$$

graduated cylinder (tenths) + burette (hundredths)

A burette adds 1.66 mL to the 108.2 mL already in a flask. A common first attempt: 1.66 has the fewest sig figs, three, so report 110. mL. Test it.

✓ Worked example 4: solution

$108.2 \text{ mL} + 1.66 \text{ mL}$
graduated cylinder (tenths) + burette (hundredths)

A common first attempt

$108.2 + 1.66 = 109.86 \rightarrow 110. \text{ mL}$
the multiplication rule applied to a sum ✗

Round to the least precise decimal place

$108.2 + 1.66 = 109.86 \rightarrow 109.9 \text{ mL}$
rounded to the tenths, the least precise place given ✓

 **Sense check:** Four sig figs survive even though one input carried three. Addition sets the answer's last decimal place; the sig-fig count follows from it.

✓ Worked example 4: the route on the map

$$108.2 \text{ mL} + 1.66 \text{ mL} = 109.9 \text{ mL}$$

found: 109.9 mL · rounded to the tenths, set by 108.2

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

🧠 **Sense check:** A sum takes the place rule, not the sig-fig count. The digit tests in the top row never decide it. ✓

Where the answer stops: the cutoff line

$$\begin{array}{r} 108.2 \\ + \quad 1.6 \\ \hline 109.8 \end{array}$$

the cutoff

108.2 stops at the tenths –
so the answer stops there too

→ 109.9 mL

rounded to the tenths: the last column left of the line

Stack the numbers at the decimal point. The least precise input draws a vertical line; nothing right of the line survives into the answer.



Practice 4

23.06 °C, then 21.4 °C

given: 23.06 °C on a digital probe · 21.4 °C on a glass thermometer · wanted: temperature change

Warm water reads **23.06 °C** on a digital probe. After cooling, a glass thermometer reads **21.4 °C**. What is the temperature change, in °C?

- A. 1.7
- B. −1.7
- C. −1.66
- D. −1.6

✓ Practice 4 · answer: B

$$21.4\text{ }^{\circ}\text{C} - 23.06\text{ }^{\circ}\text{C} = -1.66\text{ }^{\circ}\text{C} \rightarrow -1.7\text{ }^{\circ}\text{C} \text{ (answer B)}$$

change = final – initial · 21.4 ends at the tenths → round to the tenths

A subtracted backwards, initial minus final: $+1.7\text{ }^{\circ}\text{C}$ reads as warming. C used the multiplication rule: 21.4 has three sig figs, so -1.66 . A difference keeps places, not counts. D cut off the 6 instead of rounding: a first dropped digit of 6 rounds the tenths up.

 **Sense check:** The water cooled, so the change is negative. The glass thermometer reads only to the tenths, and so does the change. ✓

✓ Practice 4: the route on the map

$$21.4\text{ }^{\circ}\text{C} - 23.06\text{ }^{\circ}\text{C} = -1.7\text{ }^{\circ}\text{C}$$

found: $-1.7\text{ }^{\circ}\text{C}$ · rounded to the tenths, set by 21.4

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

🧠 **Sense check:** The path of worked example 4: a difference takes the place rule, then rounds once. ✓



Mixed operations: one rule per step

1. **Order of operations** decides the steps.
2. Each step's **own rule** decides what that intermediate is entitled to.
3. **Carry the digits unrounded**; round once, at the end, to the tightest entitlement.



Worked example 5: two rules in one calculation

$$(25.462 \text{ g} - 25.1 \text{ g}) \div 4.4 \text{ mL}$$

mass by difference from two balances · volume from a graduated cylinder

A vial is weighed full on an analytical balance, then empty on a coarser one; the sample it held is made up to 4.4 mL. Compute the mass per milliliter: one rule per step.

✓ Worked example 5: solution

$$(25.462 \text{ g} - 25.1 \text{ g}) \div 4.4 \text{ mL}$$

subtraction runs first: order of operations sets the steps

Subtract, and mark the entitlement

$$25.462 - 25.1 = 0.362 \text{ g} \rightarrow \text{entitled to the tenths (0.4-level)}$$

25.1 ends at the tenths $\rightarrow$ the difference holds one sig fig

Divide, then round once

$$0.362 \div 4.4 = 0.0823 \text{ g/mL (calculator)} \rightarrow 0.08 \text{ g/mL}$$

rounded once, at the end, to 1 sig fig, the tightest entitlement

 **Sense check:** Subtraction between near-equal readings destroys precision: the answer keeps one digit, though every input carried three or more.

✓ Worked example 5: the route on the map

$$(25.462 \text{ g} - 25.1 \text{ g}) \div 4.4 \text{ mL} = 0.08 \text{ g/mL}$$

found: 0.08 g/mL · 1 sig fig, set by the subtraction

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

 **Sense check:** Both rules ran, one per step: the place rule on the difference, the fewest-sig-figs rule on the division, one rounding at the end. ✓

Your turn: add three masses

$$14.55 \text{ g} + 0.322 \text{ g} + 2.1 \text{ g}$$

three balances, three precisions: where does the cutoff line fall?

Calculator: _____ → reported: _____

$$14.55 + 0.322 + 2.1 = 16.972 \rightarrow 17.0 \text{ g}$$

2.1 ends at the tenths → draw the cutoff line after the tenths



Sense check: Everything right of the line was never measured in the 2.1 g reading.

 Your turn: multiply

$$3.10 \times 4.520$$

two measured values: count each factor's sig figs first

Calculator: _____ → reported: _____

$$3.10 \times 4.520 = 14.012 \rightarrow 14.0$$

fewest sig figs among the factors: three, from 3.10

 **Sense check:** The trailing zero in 14.0 counts: it is the third significant figure the 3.10 supports.

Your turn: subtract, then divide

$$(6.77 - 6.2) \div 2.33$$

subtraction first, division second: each step under its own rule

Difference: _____ $\rightarrow$ entitlement: _____ $\rightarrow$ reported: _____

$$6.77 - 6.2 = 0.57 \rightarrow \text{entitled to the tenths (0.6-level)}$$

one sig fig territory: carry 0.57 unrounded into the division

$$0.57 \div 2.33 = 0.2446 \rightarrow 0.2$$

rounded once, to 1 sig fig: the subtraction set the limit

 **Sense check:** The three sig figs in 2.33 never mattered; the subtraction's one-digit entitlement caps the answer.



Practice 5

salt: 20.07 g and 1.9 g, from two balances
all of it dissolved to make 150.0 mL of solution

Two portions of salt, **20.07 g** and **1.9 g**, are dissolved together to make **150.0 mL** of solution. What concentration of salt, in g/mL, should be reported?

- A. 0.147
- B. 0.15
- C. 0.146
- D. 22.0
- E. 0.146467

✓ Practice 5 · answer: C

$20.07 + 1.9 = 21.97 \text{ g} \rightarrow$ entitled to the tenths: 3 sig figs

sum: place rule, set by 1.9 · carry 21.97 unrounded

$21.97 \div 150.0 = 0.146467 \rightarrow 0.146 \text{ g/mL}$ (answer C)

division: fewest sig figs, 3 from the sum vs 4 from 150.0

A rounded the sum to 22.0 before dividing: $22.0 \div 150.0 = 0.1467 \rightarrow 0.147$. B used one rule throughout: 1.9 has 2 sig figs, so 0.15. D stopped at the sum: 22.0 g is the salt, not the salt per milliliter. E kept every calculator digit.

🧠 **Sense check:** The sum rule, not the sig-fig count of 1.9, sets the limit: 21.97 keeps three digits. ✓

✓ Practice 5: the route on the map

$$(20.07 \text{ g} + 1.9 \text{ g}) \div 150.0 \text{ mL} = 0.146 \text{ g/mL}$$

found: 0.146 g/mL · 3 sig figs, set by the sum

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

dropped digit below 5: keep
dropped digit 5 or more: round up

 **Sense check:** Place rule on the sum, fewest sig figs on the division. 150.0 carries four: the decimal point makes its trailing zeros count. ✓



Practice 6

empty beaker 49.54 g · beaker with sample 52.4 g
the sample is then dissolved to 25.0 mL of solution

An empty beaker reads **49.54 g** on one balance. With a sample of sugar in it, a coarser balance reads **52.4 g**. The sample is dissolved in water to a final volume of **25.0 mL**. Find the mass of sugar per milliliter of solution, in g/mL.

- A. 0.12
- B. 2.9
- C. 0.114
- D. 8.7
- E. 0.11

✓ Practice 6 · answer: E

$$52.4 - 49.54 = 2.86 \text{ g} \rightarrow \text{entitled to the tenths: 2 sig figs}$$

difference: place rule, set by 52.4 · carry 2.86 unrounded

$$2.86 \div 25.0 = 0.1144 \rightarrow 0.11 \text{ g/mL (answer E)}$$

division: fewest sig figs, 2 from the difference vs 3 from 25.0

A rounded the difference before dividing: $2.9 \div 25.0 = 0.116 \rightarrow 0.12$. B stopped at the sample: 2.9 g of sugar, not grams per milliliter. C used the count rule on the subtraction: 52.4 has three sig figs, so $2.86 \div 25.0 \rightarrow 0.114$. D flipped the division: $25.0 \div 2.86 = 8.7$ mL per gram.

 **Sense check:** The coarser balance ends at the tenths, so the sample mass holds two sig figs, and so does the answer. ✓

✓ Practice 6: the route on the map

$$(52.4 \text{ g} - 49.54 \text{ g}) \div 25.0 \text{ mL} = 0.11 \text{ g/mL}$$

found: 0.11 g/mL · 2 sig figs, set by the subtraction

COUNT one number: test each digit

exact number counted · defined unlimited	nonzero digit 4.3 · 27 counts	captive zero 403 · 2.05 counts	leading zero 0.04 · 0.0072 never counts	trailing zero decimal point: 4.20 counts	trailing zero no decimal point: 420 placeholder
--	---	--	---	--	---

REPORT a calculated result

count each factor

× or ÷ → **fewest sig figs**

+ or – → **least precise place**

both in one problem:
one rule per step

round once, at the end

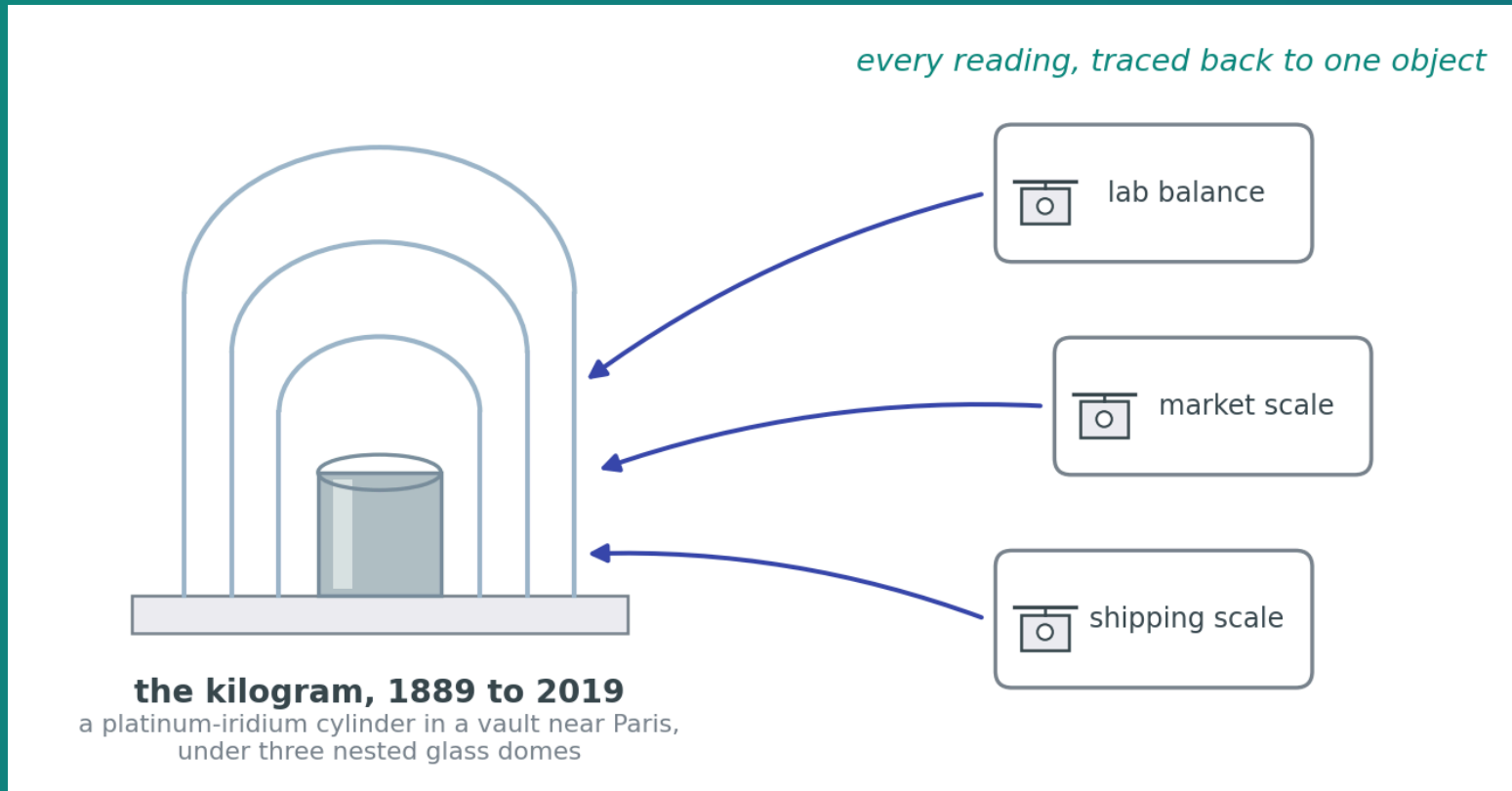
dropped digit below 5: keep
dropped digit 5 or more: round up

 **Sense check:** The path of worked example 5: place rule on the difference, fewest sig figs on the division, one rounding at the end. ✓

3 • The SI Unit System

Report every measurement as a number plus a unit, name the five SI base units chemistry uses, and read working and built units like the gram, the milliliter, and g/mL as combinations of them.

One object was the kilogram



From 1889 to 2019, the kilogram was a platinum-iridium cylinder near Paris. Every balance on Earth traced to that one object. Three glass domes kept off the dust.

Every measurement reports three things

21.6 mL

number: 21.6 · unit: mL · uncertainty: the last digit, 6, is the estimate

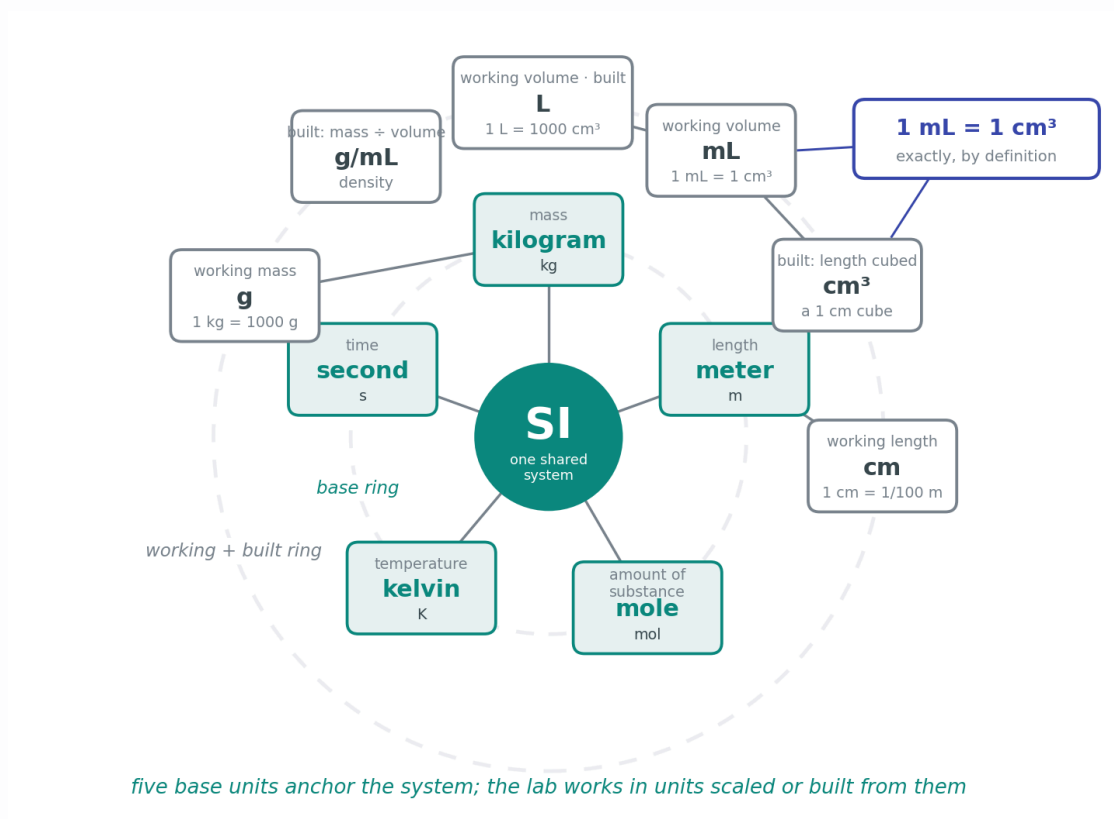
A measurement states how much, against what standard, and how certain. The number gives the size. The unit names the standard. The last digit carries the uncertainty. Without the unit, the rest means nothing.

A number means nothing without its unit

500 → not a measurement · 500 mg → one aspirin tablet
the unit names what was counted

Two of the three are the number and the unit. The number tells how much; the unit tells of what. Strip the unit and the number carries no information another scientist can use.

One system, five base units



The SI system gives every quantity one agreed base unit. Chemistry leans on five: the meter, the kilogram, the second, the kelvin, and the mole. Results measured anywhere compare directly.

Working units of the lab

$$1 \text{ kg} = 1000 \text{ g} \cdot 1 \text{ L} = 1000 \text{ mL}$$

balances read grams · glassware reads milliliters

A kilogram of reagent rarely sits on a bench. Balances read grams and glassware reads milliliters; each working unit ties to a base unit by a power of ten.

The liter is a built unit

$$10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 1000 \text{ cm}^3 = 1 \text{ L}$$

$$1 \text{ mL} = 1 \text{ cm}^3 \text{ exactly} \cdot 1 \text{ L} = 1000 \text{ mL}$$

Multiplying base units builds new ones. A cube 10 cm on each edge encloses one liter, and the milliliter and the cubic centimeter name the same volume exactly.

Density is a built unit

$$d = m / V \rightarrow \text{grams per milliliter} \rightarrow \text{g/mL}$$

two measurements combine into one new unit

Dividing a mass by a volume builds a new unit, the gram per milliliter. The unit itself states the meaning: the grams packed into each milliliter of a substance.

Practice 1

base units: m · kg · s · K · mol

wanted: the base unit for one measured quantity

A pharmacist weighs a powder sample. Which SI base unit matches the quantity being measured?

- A. the kilogram (kg)
- B. the gram (g)
- C. the liter (L)
- D. the meter (m)

✓ Practice 1 · answer: A

weighing → mass → kilogram (kg) (answer A)
the one base unit whose name carries a prefix

B is the working unit of the bench, not the base unit; 1 kg = 1000 g ties every gram reading to the kilogram. C measures volume, the space a sample fills. D measures length.

🧠 **Sense check:** Name the quantity first, then match the unit: a balance measures mass, and mass takes the kilogram.

✓



Practice 2

four notebook entries: a reading and a note on its unit
wanted: the entry that is right on every count

Which notebook entry is correct?

- A. one paper clip: 1 kg, recorded in the SI base unit of mass
- B. a warm water bath: 310 K, recorded in a built unit
- C. a flask of solution: 250 m, recorded as its volume
- D. a sugar solution: 1.20 g/mL, recorded in a built unit

✓ Practice 2 · answer: D

1.20 g/mL: a density, grams divided by milliliters (answer D)
the unit matches the quantity · the size fits a solution · built from two units

A has the right unit and an absurd size: 1 kg is a bag of sugar, and a paper clip is about a gram. B misnames the kelvin; it is one of the five base units. C reports a length; a volume takes mL or L.

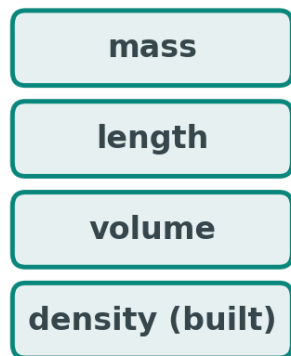
 **Sense check:** Three tests per entry: the unit matches the quantity, the number fits the object, and base or built is named right. ✓



The method

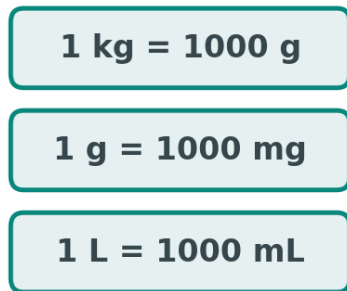
1. **Name the quantity:** mass, length, volume, or a built quantity.
2. **Find the SI equality:** a prefix or a build.
3. **Write it as a factor:** the given unit cancels.
4. **Check the size:** a smaller unit gives a larger count.

1 · Name the quantity

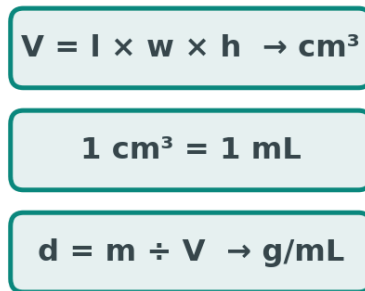


2 · Find the SI equality

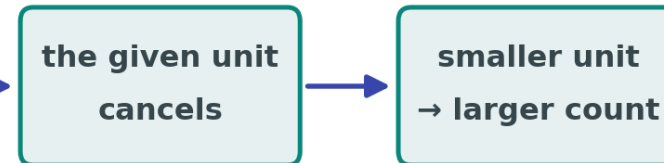
prefix: a power of ten



build: from other units



3 · Write it as a factor 4 · Check the size





Guided example: liters to milliliters

$$1 \text{ L} = 1000 \text{ mL}$$

given: 1.89 L · wanted: mL

A carton of milk holds **1.89 L**. How many milliliters does it hold?

Name the quantity first, then find the SI equality that links the two units.

✓ Guided example: solution

$$1 \text{ L} = 1000 \text{ mL}$$

given: 1.89 L · wanted: mL

Step 1 · Name the quantity

Step 2 · Find the SI equality

Step 3 · Write it as a factor

$$1.89 \text{ L} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 1.89 \times 10^3 \text{ mL}$$

Step 4 · Check the size

 **Sense check:** A milliliter is a thousandth of a liter, so the count grows a thousandfold: 1.89 → 1890. Three sig figs stay: $1.89 \times 10^3 \text{ mL}$ ✓

✓ Guided example: the route on the map

$$1 \text{ L} = 1000 \text{ mL}$$

given: 1.89 L · found: $1.89 \times 10^3 \text{ mL}$

1 · Name the quantity



2 · Find the SI equality

prefix: a power of ten

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ g} = 1000 \text{ mg}$$

$$1 \text{ L} = 1000 \text{ mL}$$

build: from other units

$$V = l \times w \times h \rightarrow \text{cm}^3$$

$$1 \text{ cm}^3 = 1 \text{ mL}$$

$$d = m \div V \rightarrow \text{g/mL}$$

3 · Write it as a factor 4 · Check the size

the given unit
cancels

smaller unit
→ larger count

🧠 **Sense check:** Volume to volume takes one prefix equality, $1 \text{ L} = 1000 \text{ mL}$. No build was needed. ✓



Practice 3

18 mg of iron per tablet

given: 18 mg · wanted: g

A multivitamin tablet lists **18 mg** of iron. How many grams of iron is that?

A. 1.8×10^4

B. 0.18

C. 0.018

D. 18

✓ Practice 3 · answer: C

1 g = 1000 mg
given: 18 mg · wanted: g

$$18 \text{ mg} \times \frac{1 \text{ g}}{1000 \text{ mg}} = 0.018 \text{ g (answer C)}$$

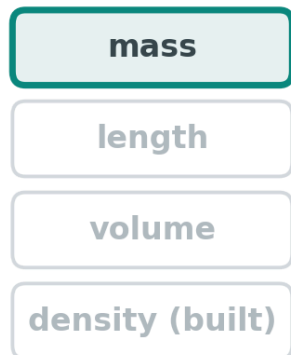
A flipped the factor: $18 \times 1000 = 1.8 \times 10^4$, a count that grew while the unit got larger. B used the centi power, $1 \text{ g} = 100 \text{ mg}$: $18 \div 100 = 0.18$. D relabeled the unit without converting: 18 mg is not 18 g.

 **Sense check:** A gram holds 1000 mg, so the count in grams is a thousand times smaller: $18 \rightarrow 0.018$. Two sig figs stay ✓

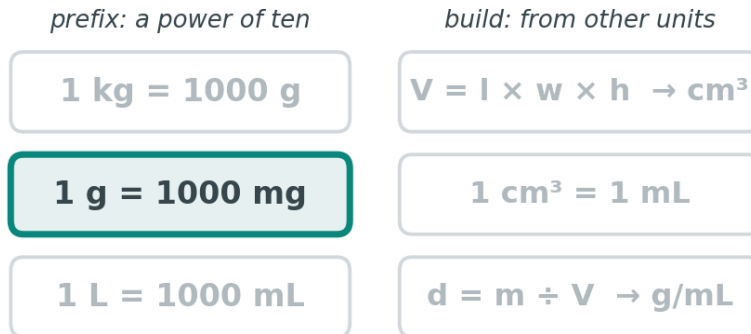
✓ Practice 3: the route on the map

$1 \text{ g} = 1000 \text{ mg}$
given: 18 mg · found: 0.018 g

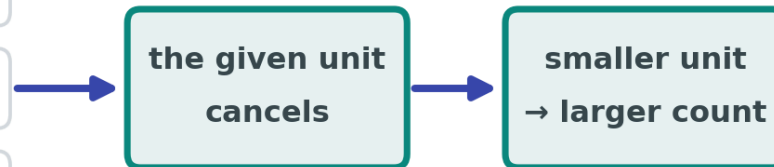
1 · Name the quantity



2 · Find the SI equality



3 · Write it as a factor 4 · Check the size



 **Sense check:** Mass to mass takes one prefix equality, $1 \text{ g} = 1000 \text{ mg}$, the path of the guided example on a different quantity. ✓



Practice 4

a cube, 1.6 cm on each edge

given: 1.6 cm edge · wanted: mL

A sugar cube measures **1.6 cm** on each edge. What volume does it fill, in milliliters?

A. 4.1

B. 2.6

C. 4.8

D. 4.1×10^{-3}

✓ Practice 4 · answer: A

$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL}$$

given: 1.6 cm edge · wanted: mL

Two moves: build the volume, then apply the exact identity.

$$V = 1.6 \text{ cm} \times 1.6 \text{ cm} \times 1.6 \text{ cm} = 4.1 \text{ cm}^3 \times \frac{1 \text{ mL}}{1 \text{ cm}^3} = \mathbf{4.1 \text{ mL (answer A)}}$$

B squared the edge: $1.6 \times 1.6 = 2.6 \text{ cm}^2$, the area of one face. C multiplied the edge by 3: $1.6 \times 3 = 4.8 \text{ cm}$, still a length. D converted to liters: $4.1 \text{ cm}^3 \times (1 \text{ L} / 1000 \text{ cm}^3) = 4.1 \times 10^{-3} \text{ L}$, not mL.

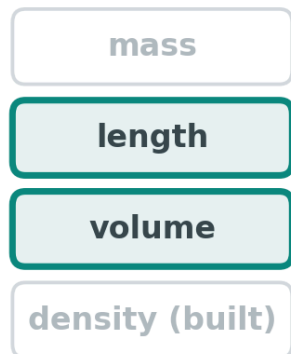
 **Sense check:** A sugar cube is a little larger than a 1 cm cube, so it fills a few milliliters: 4.1 mL. The edge has two sig figs, so the volume keeps two ✓

✓ Practice 4: the route on the map

$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL}$$

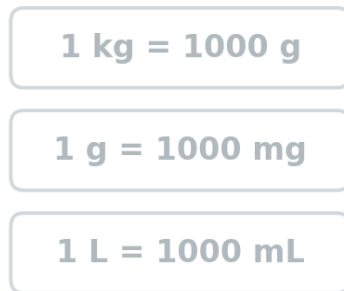
given: 1.6 cm edge · found: 4.1 mL

1 · Name the quantity

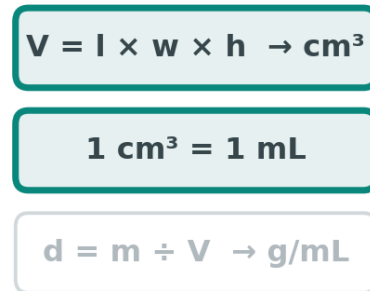


2 · Find the SI equality

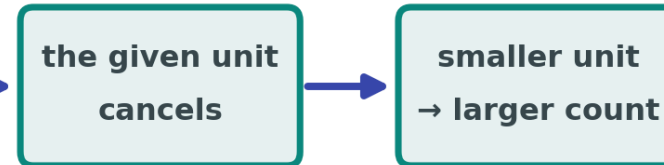
prefix: a power of ten



build: from other units



3 · Write it as a factor 4 · Check the size



 **Sense check:** The givens are lengths and the wanted quantity is a volume, so both equalities are builds: length cubed, then $1 \text{ cm}^3 = 1 \text{ mL}$. No prefix was needed. ✓



Practice 5

a bin $40.0\text{ cm} \times 30.0\text{ cm} \times 25.0\text{ cm}$
given: three edges in cm · wanted: L

A plastic storage bin is **40.0 cm** long, **30.0 cm** wide, and **25.0 cm** deep. How many liters does it hold?

- A. 3.00×10^4
- B. 0.0300
- C. 3.00×10^7
- D. 30.0

✓ Practice 5 · answer: D

$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL} \cdot 1 \text{ L} = 1000 \text{ mL}$$

given: 40.0 cm, 30.0 cm, 25.0 cm · wanted: L

$$V = 40.0 \text{ cm} \times 30.0 \text{ cm} \times 25.0 \text{ cm} = 3.00 \times 10^4 \text{ cm}^3 = 3.00 \times 10^4 \text{ mL}$$

$$3.00 \times 10^4 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 30.0 \text{ L (answer D)}$$

A stopped at the built volume: 3.00×10^4 is in mL, not L. B converted the edges to meters: $0.400 \times 0.300 \times 0.250 = 0.0300 \text{ m}^3$, and one cubic meter holds 1000 L. C flipped the liter factor: $3.00 \times 10^4 \times 1000 = 3.00 \times 10^7$.

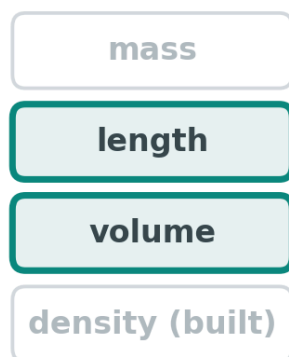
🧠 **Sense check:** A liter is a 10 cm cube. The bin's edges hold 4, 3, and 2.5 of those cubes: $4 \times 3 \times 2.5 = 30.0 \text{ L}$ ✓

✓ Practice 5: the route on the map

$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL} \cdot 1 \text{ L} = 1000 \text{ mL}$$

given: 40.0 cm, 30.0 cm, 25.0 cm · found: 30.0 L

1 · Name the quantity



2 · Find the SI equality

prefix: a power of ten

$$1 \text{ kg} = 1000 \text{ g}$$

$$1 \text{ g} = 1000 \text{ mg}$$

$$1 \text{ L} = 1000 \text{ mL}$$

build: from other units

$$V = l \times w \times h \rightarrow \text{cm}^3$$

$$1 \text{ cm}^3 = 1 \text{ mL}$$

$$d = m \div V \rightarrow \text{g/mL}$$

3 · Write it as a factor 4 · Check the size

the given unit
cancels

smaller unit
→ larger count

 **Sense check:** The path of Practice 4, one step longer: the built volume lands in mL, and the prefix equality 1 L = 1000 mL carries it to liters. ✓



Practice 6

an acrylic block $5.00\text{ cm} \times 4.00\text{ cm} \times 2.50\text{ cm}$ · mass 0.0590 kg

given: three edges and a mass · wanted: density, in g/mL

A block of acrylic plastic measures $5.00\text{ cm} \times 4.00\text{ cm} \times 2.50\text{ cm}$ and has a mass of 0.0590 kg . What is its density, in g/mL?

- A. 0.847
- B. 1.18×10^3
- C. 0.00118
- D. 2.95
- E. 1.18

✓ Practice 6 · answer: E

$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL} \cdot 1 \text{ kg} = 1000 \text{ g} \cdot d = m \div V$$

given: $5.00 \text{ cm} \times 4.00 \text{ cm} \times 2.50 \text{ cm} \cdot 0.0590 \text{ kg} \cdot \text{wanted: g/mL}$

$$V = 5.00 \text{ cm} \times 4.00 \text{ cm} \times 2.50 \text{ cm} = 50.0 \text{ cm}^3 = 50.0 \text{ mL} \cdot 0.0590 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 59.0 \text{ g}$$

$$d = \frac{59.0 \text{ g}}{50.0 \text{ mL}} = 1.18 \text{ g/mL (answer E)}$$

A flipped the ratio: $50.0 \text{ mL} \div 59.0 \text{ g} = 0.847 \text{ mL/g}$. B divided by liters: $59.0 \text{ g} \div 0.0500 \text{ L} = 1.18 \times 10^3 \text{ g/L}$. C skipped kg $\rightarrow$ g: $0.0590 \div 50.0 = 0.00118 \text{ kg/mL}$. D divided by one face: $59.0 \div (5.00 \times 4.00) = 2.95 \text{ g/cm}^2$.

 **Sense check:** Acrylic sinks in water, 1.00 g/mL , but only just: 1.18 g/mL . Every given has three sig figs ✓

✓ Practice 6: the route on the map

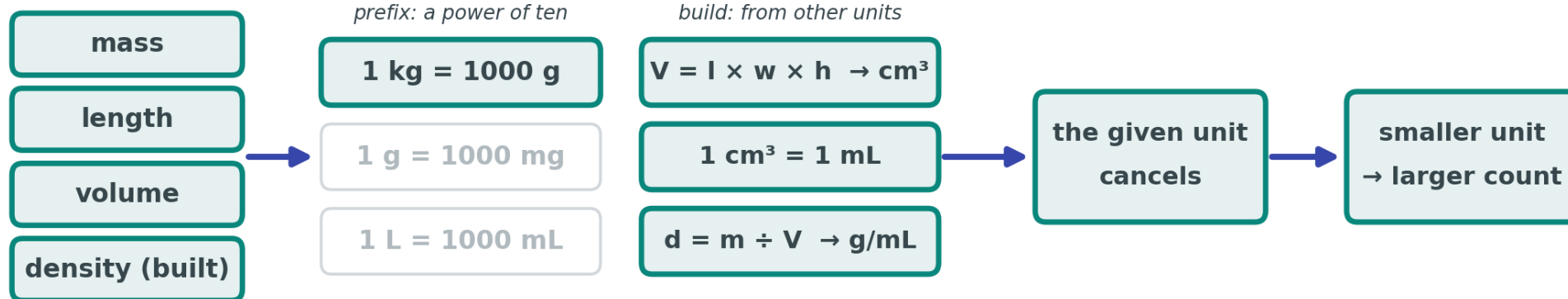
$$V = l \times w \times h \cdot 1 \text{ cm}^3 = 1 \text{ mL} \cdot 1 \text{ kg} = 1000 \text{ g} \cdot d = m \div V$$

given: $5.00 \text{ cm} \times 4.00 \text{ cm} \times 2.50 \text{ cm} \cdot 0.0590 \text{ kg}$ · found: 1.18 g/mL

1 · Name the quantity

2 · Find the SI equality

3 · Write it as a factor 4 · Check the size



 **Sense check:** Three builds and one prefix: the volume from three lengths, cm^3 renamed as mL, grams from kilograms, then density as mass over volume. ✓

4 · Dimensional Analysis

Convert a measurement into any unit by chaining conversion factors, each one picked so the unit before it cancels.

Same number, wrong unit

same numbers, different units

the ground team reported

lbf·s · **pound-seconds**

≠

the navigation software read

N·s · **newton-seconds**

$$1 \text{ lbf} \cdot \text{s} = 4.448 \text{ N} \cdot \text{s}$$

the flyby that resulted

planned pass · 226 km



actual pass · 57 km

atmosphere

Mars surface

The Mars Climate Orbiter was lost in 1999. One team reported thruster impulse in **pound-seconds** and the software read **newton-seconds**. It flew too low and broke apart.

Old unit to new unit

$$1 \text{ dozen} = 12 \text{ donuts}$$

$$\text{one amount, two names} \cdot \text{old unit} \times (\text{new unit} / \text{old unit}) = \text{new unit}$$

A box holds 2 dozen donuts. The equality converts dozens to donuts:

$$2 \text{ ~~dozen~~} \times \frac{12 \text{ donuts}}{1 \text{ ~~dozen~~}} = \mathbf{24 \text{ donuts}}$$

How many dozen are 36 donuts?

$$36 \text{ ~~donuts~~} \times \frac{1 \text{ dozen}}{12 \text{ ~~donuts~~}} = \mathbf{3 \text{ dozen}}$$

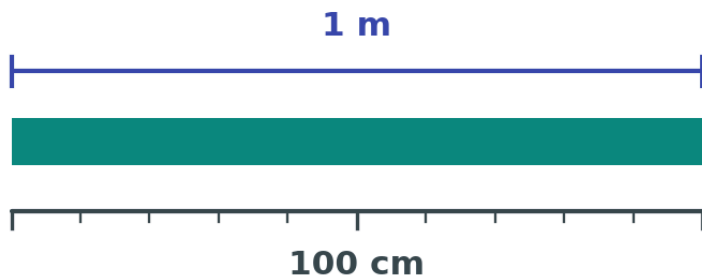
Old unit on the bottom, new unit on top. Every conversion works this way.

A conversion factor equals 1

$$1 \text{ m} = 100 \text{ cm}$$

one length, two names

An equality names one amount two ways. Written as a fraction, top matches bottom, so the fraction equals 1. Multiplying by 1 changes the unit, never the quantity.



one length, two names

$$\frac{1 \text{ m}}{100 \text{ cm}} = 1$$

$$\frac{100 \text{ cm}}{1 \text{ m}} = 1$$

top and bottom name
the same length

Units cancel like symbols in algebra

A unit on top cancels the same unit below. The right factor removes the given unit and leaves the wanted one.

$$\text{given } \cancel{\text{unit A}} \times \frac{\text{wanted unit B}}{\cancel{\text{unit A}}} = \text{answer, in unit B}$$

Metric prefixes are equalities

prefix	symbol	value
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1

prefix	symbol	value
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

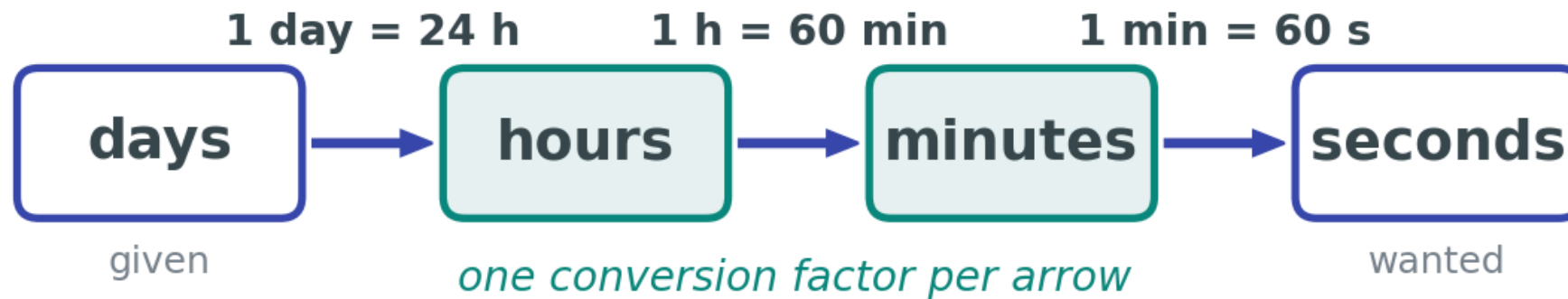
shaded rows appear in almost every problem

Each prefix defines an equality with any base unit: $1 \text{ km} = 10^3 \text{ m}$, $1 \text{ mg} = 10^{-3} \text{ g}$, $1 \text{ ms} = 10^{-3} \text{ s}$. One table covers every metric conversion.

King Henry Died By Drinking Chocolate Milk

kilo · hecto · deka · base · deci · centi · milli: one power of ten per word · beyond this run, the table steps by thousands

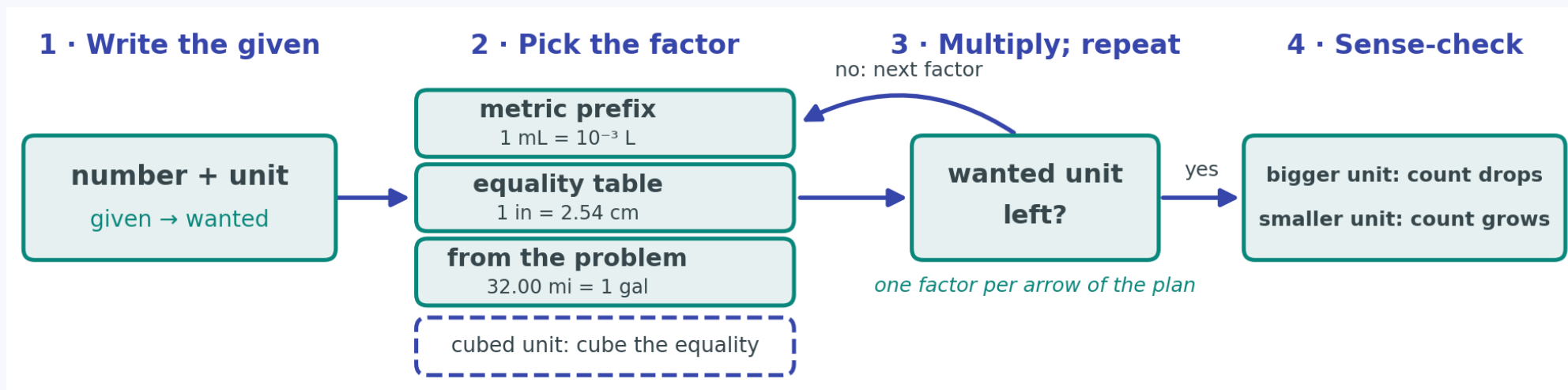
The unit plan



Sketch the route from the given unit to the wanted unit before any arithmetic. One conversion factor per arrow. When no single equality links them, the route runs through units in between.

The method

1. **Write the given:** number and unit.
2. **Pick the factor that cancels its unit:** the given unit goes in the denominator.
3. **Multiply; repeat until the wanted unit survives.**
4. **Sense-check** the size and the surviving unit.





Guided example: a newborn's mass

Step 1 • Write the given

$$1 \text{ kg} = 2.205 \text{ lb}$$

given: 7.50 lb • wanted: kg

A newborn weighs **7.50 lb**. The hospital chart records mass in kilograms. Express the baby's mass in kilograms.

The old unit is lb. The new unit is kg.

✓ Guided example: solution

$$1 \text{ kg} = 2.205 \text{ lb}$$

given: 7.50 lb · wanted: kg

Step 2 · Pick the factor that cancels its unit

Step 3 · Multiply; repeat until the wanted unit survives

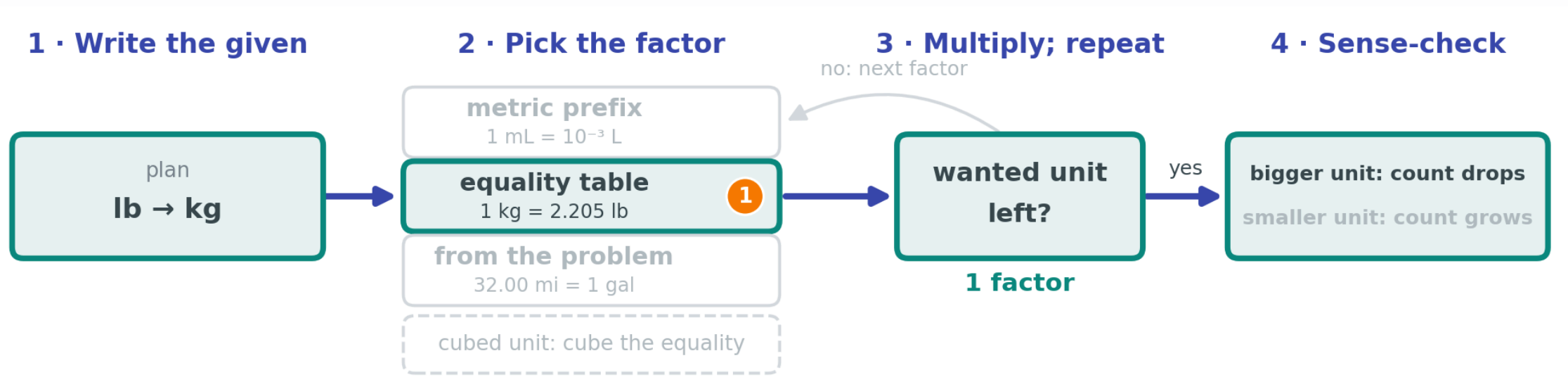
$$7.50 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} = 3.40 \text{ kg}$$

Step 4 · Sense-check

 **Sense check:** A kilogram is bigger than a pound, so the count in kilograms is smaller: 7.50 → 3.40. A newborn of about 3.4 kg is typical. ✓

✓ Guided example: the route on the map

1 kg = 2.205 lb
given: 7.50 lb · found: 3.40 kg



Sense check: One factor, from the equality table. kg survived after one factor, so the chain did not repeat. kg is the bigger unit, so the count dropped. ✓



Practice 1

$$1 \text{ in} = 2.54 \text{ cm}$$

given: 16.0 in · wanted: cm

A large pizza measures **16.0 in** across. How many centimeters across is it?

- A. 6.30
- B. 406
- C. 16.0
- D. 40.6

✓ Practice 1 · answer: D

$$1 \text{ in} = 2.54 \text{ cm}$$

given: 16.0 in · wanted: cm

$$16.0 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 40.6 \text{ cm (answer D)}$$

A flipped the factor: $16.0 / 2.54 = 6.30$. B used $1 \text{ in} = 25.4 \text{ cm}$, the millimeter number: $16.0 \times 25.4 = 406$. C relabeled the unit without converting: 16.0. The 2.54 is exact by definition, so the three sig figs of 16.0 carry through.

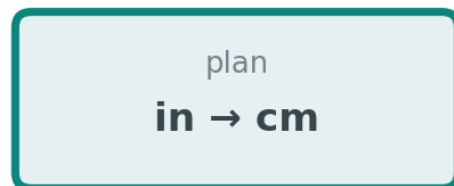
🧠 **Sense check:** A centimeter is smaller than an inch, so the count grows by 2.54: $16.0 \rightarrow 40.6$. ✓

✓ Practice 1: the route on the map

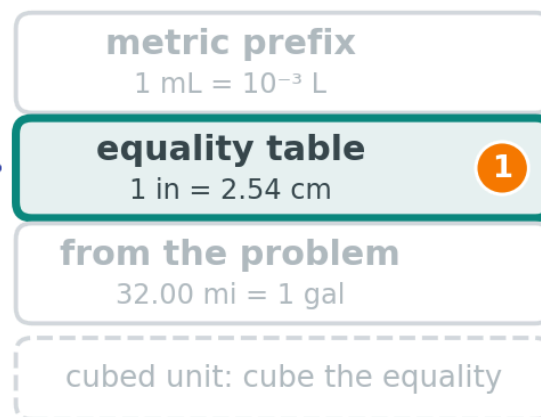
$$1 \text{ in} = 2.54 \text{ cm}$$

given: 16.0 in · found: 40.6 cm

1 · Write the given

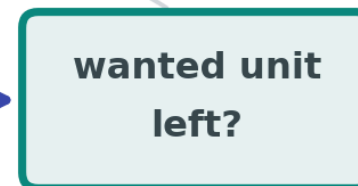


2 · Pick the factor

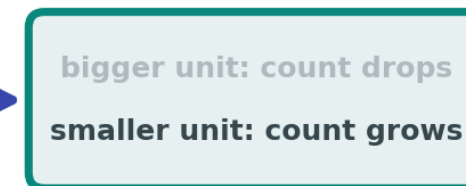


3 · Multiply; repeat

no: next factor



4 · Sense-check



 **Sense check:** One factor, from the equality table, with inches on the bottom. cm is the smaller unit, so the count grew. ✓



Worked example 1

Step 1 • Write the given

$$1 \text{ m} = 100 \text{ cm}$$

given: 347 cm • wanted: m

A whiteboard measures **347 cm** across. Express the width in meters.

A common first attempt uses the factor written 100 cm over 1 m. Test it.

✓ Worked example 1: solution

1 m = 100 cm
given: 347 cm · wanted: m

A common first attempt

$$347 \text{ cm} \times \frac{100 \text{ cm}}{1 \text{ m}} = 34,700 \text{ cm}^2/\text{m} \times$$

Step 2 · Pick the factor that cancels its unit

Step 3 · Multiply; repeat until the wanted unit survives

$$347 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 3.47 \text{ m}$$

 **Sense check:** A meter holds 100 cm, so the count in meters must be smaller: 347 → 3.47. The width itself is unchanged. ✓

✓ Worked example 1: the route on the map

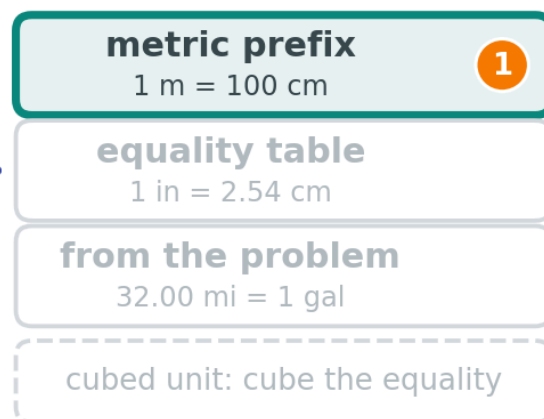
$$1 \text{ m} = 100 \text{ cm}$$

given: 347 cm · found: 3.47 m

1 · Write the given

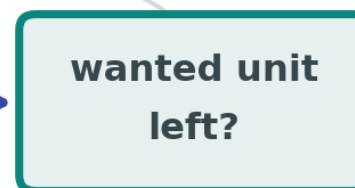


2 · Pick the factor

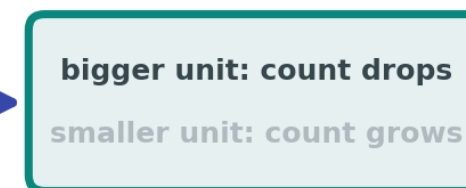


3 · Multiply; repeat

no: next factor



4 · Sense-check



 **Sense check:** One prefix factor, with cm on the bottom. m is the bigger unit, so the count dropped. ✓



Worked example 2

Step 1 • Write the given

$$1 \mu\text{m} = 10^{-6} \text{ m} \cdot 1 \text{ mm} = 10^{-3} \text{ m}$$

given: $45 \mu\text{m}$ • wanted: mm

A fine human hair measures **$45 \mu\text{m}$** across. Express the width in millimeters.

No single equality links μm to mm. The unit plan runs through the base unit: $\mu\text{m} \rightarrow \text{m} \rightarrow \text{mm}$.

✓ Worked example 2: solution

$$1 \mu\text{m} = 10^{-6} \text{ m} \cdot 1 \text{ mm} = 10^{-3} \text{ m}$$

given: $45 \mu\text{m}$ · wanted: mm · plan: $\mu\text{m} \rightarrow \text{m} \rightarrow \text{mm}$

Step 2 · Pick the factor that cancels its unit

$$45 \cancel{\mu\text{m}} \times \frac{10^{-6} \text{ m}}{1 \cancel{\mu\text{m}}} = 4.5 \times 10^{-5} \text{ m}$$

Step 3 · Multiply; repeat until the wanted unit survives

$$45 \cancel{\mu\text{m}} \times \frac{10^{-6} \cancel{\text{m}}}{1 \cancel{\mu\text{m}}} \times \frac{1 \text{ mm}}{10^{-3} \cancel{\text{m}}} = 0.045 \text{ mm}$$

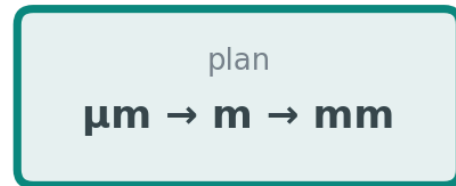
 **Sense check:** A millimeter holds 1000 μm , so the count drops by 1000: $45 \rightarrow 0.045$. A hair is thinner than a millimeter. ✓

✓ Worked example 2: the route on the map

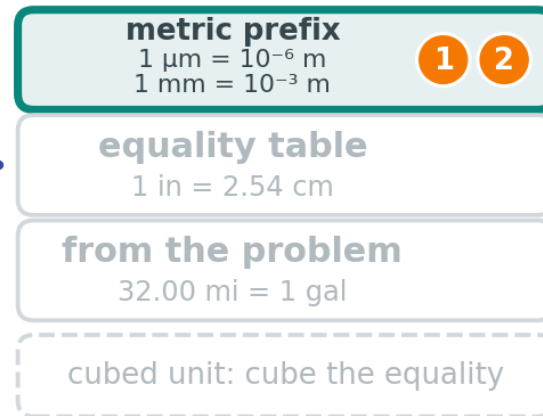
$$1 \mu\text{m} = 10^{-6} \text{ m} \cdot 1 \text{ mm} = 10^{-3} \text{ m}$$

given: $45 \mu\text{m}$ · found: 0.045 mm

1 · Write the given



2 · Pick the factor

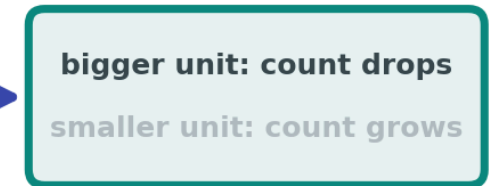


3 · Multiply; repeat

no: next factor



4 · Sense-check



🧠 **Sense check:** Two prefix factors, through the base unit. After the first, m was left, not the wanted mm, so the chain repeated once. ✓

Your turn: feet to centimeters

$$1 \text{ ft} = 12 \text{ in} \cdot 1 \text{ in} = 2.54 \text{ cm}$$

given: 6.00 ft · wanted: cm

A doorway stands **6.00 ft** tall. The unit plan: ft → in → cm.

$$6.00 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{\text{cm}}{\text{in}} = \text{cm}$$

Fill the second factor from 1 in = 2.54 cm, then compute.

$$6.00 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = \mathbf{183 \text{ cm}}$$

 Practice 2

21 mg $\rightarrow$ kg

given: 21 mg $\cdot$ wanted: kg

A grain of rice has a mass of **21 mg**. What is its mass in kilograms?

A. 2.1×10^{-5}

B. 0.021

C. 21

D. 2.1×10^7

✓ Practice 2 · answer: A

$$1 \text{ mg} = 10^{-3} \text{ g} \cdot 1 \text{ kg} = 10^3 \text{ g}$$

given: 21 mg · plan: mg → g → kg

$$21 \text{ mg} \times \frac{10^{-3} \text{ g}}{1 \text{ mg}} \times \frac{1 \text{ kg}}{10^3 \text{ g}} = 2.1 \times 10^{-5} \text{ kg (answer A)}$$

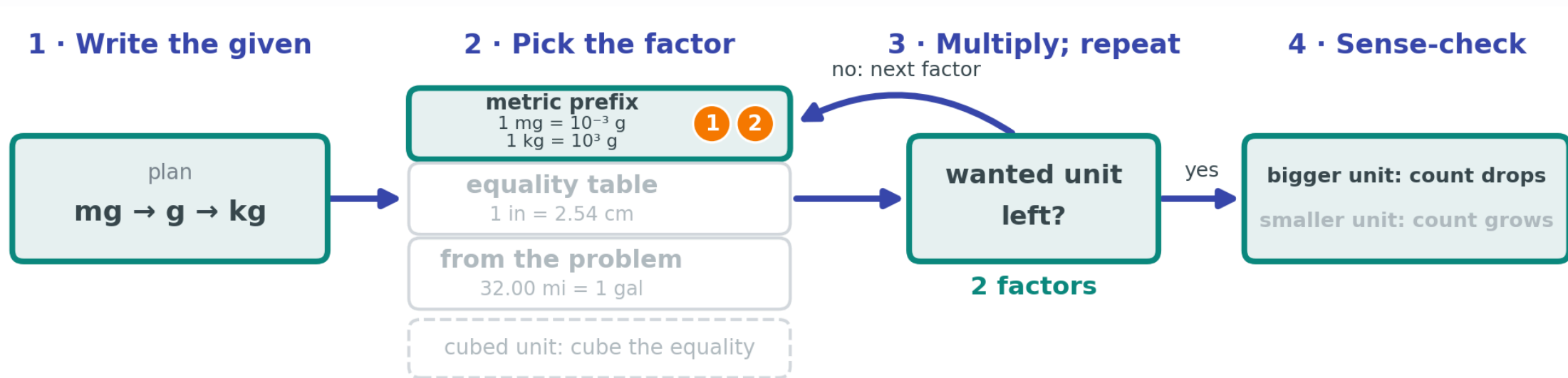
B stopped mid-plan at grams: $21 \times 10^{-3} = 0.021 \text{ g}$. C flipped the kilogram factor: $21 \times 10^{-3} \times 10^3 = 21$, the starting number again. D flipped both factors: $21 \times 10^3 \times 10^3 = 2.1 \times 10^7$.

 **Sense check:** A kilogram holds 10^6 mg, so the count drops by a million: $21 \rightarrow 2.1 \times 10^{-5}$. Prefix equalities are exact, so the two sig figs of 21 carry through. ✓

✓ Practice 2: the route on the map

$$1 \text{ mg} = 10^{-3} \text{ g} \cdot 1 \text{ kg} = 10^3 \text{ g}$$

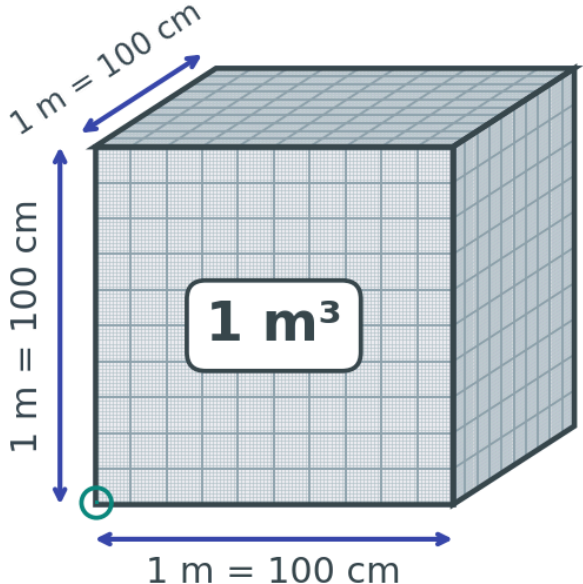
given: 21 mg · found: $2.1 \times 10^{-5} \text{ kg}$



Sense check: Two prefix factors, through the base unit, g. After the first, grams were left, not kilograms, so the chain repeated once. ✓

Cubed units: cube the whole equality

one cubic meter holds a million cubic centimeters



$(1\text{ m})^3 = (100\text{ cm})^3$

$1\text{ m}^3 = 100 \times 100 \times 100\text{ cm}^3$

$1\text{ m}^3 = 10^6\text{ cm}^3$

the number and the unit are both cubed

 one small cube = 1 cm^3

$100 \times 100 \times 100 = 1,000,000$ small cubes

Volume is length $\times$ length $\times$ length. A cubed unit takes the length equality cubed, number and unit together: $(1\text{ m})^3 = (100\text{ cm})^3$, so $1\text{ m}^3 = 10^6\text{ cm}^3$.



Worked example 3: cubic meters to cubic centimeters

Step 1 • Write the given

$$1 \text{ m} = 100 \text{ cm}$$

given: 3.5 m^3 • wanted: cm^3

A concrete pour measures 3.5 m^3 . Express the volume in cubic centimeters.

A common first attempt uses the linear factor, 100 cm over 1 m, once. Test it.

✓ Worked example 3: solution

$$1 \text{ m} = 100 \text{ cm}$$

given: 3.5 m^3 · wanted: cm^3

A common first attempt

$$3.5 \text{ m}^3 \times \frac{100 \text{ cm}}{1 \text{ m}} = 350 \text{ m}^2 \cdot \text{cm} \times$$

Step 2 · Pick the factor that cancels its unit

Step 3 · Multiply; repeat until the wanted unit survives

$$3.5 \text{ m}^3 \times \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} = 3.5 \times 10^6 \text{ cm}^3$$

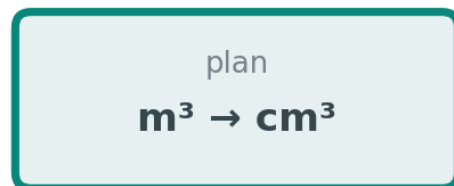
 **Sense check:** A centimeter cube is sugar-cube sized, and a million of them fill one cubic meter. The count grows by 10^6 : $3.5 \rightarrow 3,500,000$. The volume itself is unchanged. ✓

✓ Worked example 3: the route on the map

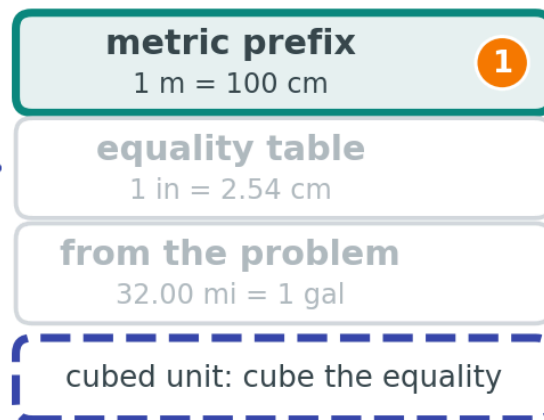
$$1 \text{ m} = 100 \text{ cm} \cdot (1 \text{ m})^3 = (100 \text{ cm})^3$$

given: 3.5 m^3 · found: $3.5 \times 10^6 \text{ cm}^3$

1 · Write the given

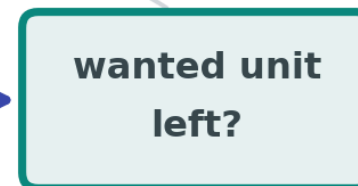


2 · Pick the factor

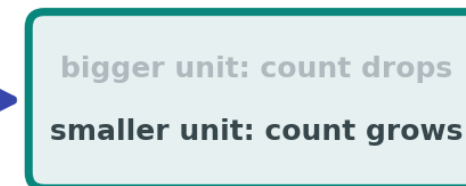


3 · Multiply; repeat

no: next factor



4 · Sense-check



🧠 **Sense check:** One prefix equality, cubed, number and unit together: $1 \text{ m}^3 = 10^6 \text{ cm}^3$. cm^3 is the smaller unit, so the count grew. ✓



Practice 3

$$1 \text{ m} = 100 \text{ cm}$$

given: $7.5 \times 10^5 \text{ cm}^3$ · wanted: m^3

An aquarium holds $7.5 \times 10^5 \text{ cm}^3$ of water. How many cubic meters does it hold?

- A. 7500
- B. 7.5×10^{11}
- C. 0.75
- D. 7.5×10^7

✓ Practice 3 · answer: C

$$1 \text{ m} = 100 \text{ cm} \cdot (1 \text{ m})^3 = (100 \text{ cm})^3$$

given: $7.5 \times 10^5 \text{ cm}^3$ · wanted: m^3

$$7.5 \times 10^5 \text{ cm}^3 \times \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} = \mathbf{0.75 \text{ m}^3 \text{ (answer C)}}$$

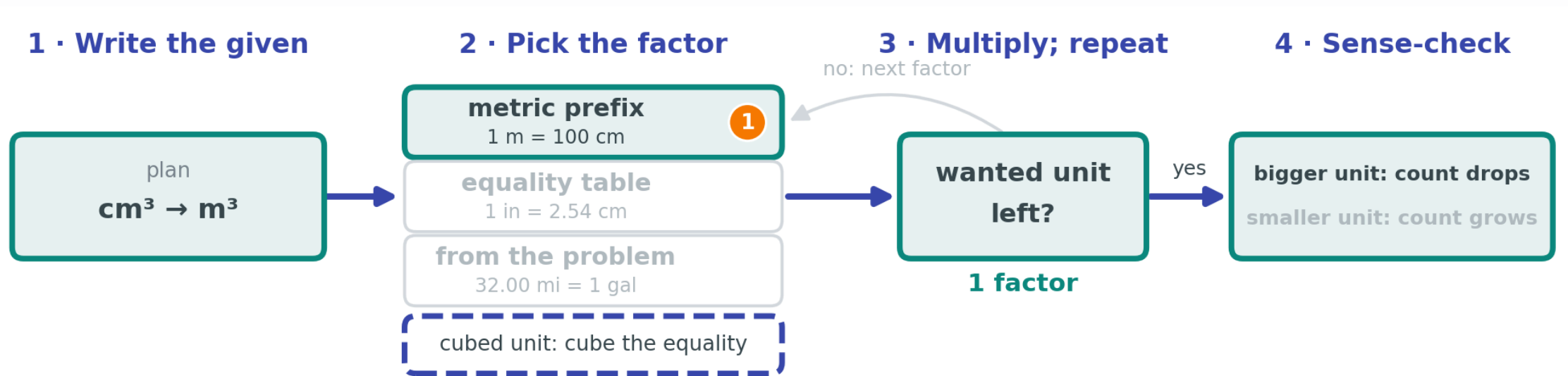
A cubed the unit without cubing the number, dividing by only 100: $7.5 \times 10^5 / 100 = 7500$.
 B pointed the cubed factor the wrong way, so nothing cancels: $7.5 \times 10^5 \times 10^6 = 7.5 \times 10^{11}$.
 D used the linear factor, inverted: $7.5 \times 10^5 \times 100 = 7.5 \times 10^7$.

 **Sense check:** A cubic meter is a cube one meter on each edge, about refrigerator size. A large aquarium is somewhat smaller: 0.75 m^3 . The count drops by 10^6 from cm^3 to m^3 . ✓

✓ Practice 3: the route on the map

$$1 \text{ m} = 100 \text{ cm} \cdot (1 \text{ m})^3 = (100 \text{ cm})^3$$

given: $7.5 \times 10^5 \text{ cm}^3$ · found: 0.75 m^3



Sense check: The same cubed prefix equality, pointed the other way: cm^3 on the bottom. m^3 is the bigger unit, so the count dropped. ✓

Where this goes wrong

Inverting the factor. $347 \text{ cm} \times (100 \text{ cm} / 1 \text{ m}) = 34,700 \text{ cm}^2/\text{m}$. No unit cancels, and the answer is not in meters. If the units do not cancel, the factor is inverted: the correct setup gives 3.47 m.

Stopping mid-plan. The plan $\mu\text{m} \rightarrow \text{m} \rightarrow \text{mm}$ has two arrows. Stopping after one gives $4.5 \times 10^{-5} \text{ m}$: meters, not the wanted millimeters. The chain ends at 0.045 mm.

Calculating without a plan. On a multi-step chain, write the route first: given unit $\rightarrow \dots \rightarrow$ wanted unit. Each arrow names the factor to pick; a skipped arrow shows up as a unit that will not cancel.

Cubing the unit without cubing the number. $1 \text{ m}^3 = 100 \text{ cm}^3$ is false. Cubing $1 \text{ m} = 100 \text{ cm}$ cubes both sides entirely: $(1 \text{ m})^3 = (100 \text{ cm})^3 = 10^6 \text{ cm}^3$. The un-cubed factor turns 3.5 m^3 into 350 cm^3 instead of $3.5 \times 10^6 \text{ cm}^3$.



Practice 4

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L}$$

given: 2500. mL · wanted: gal

A soft-drink bottle holds **2500. mL**. How many gallons does it hold?

- A. 2.500
- B. 0.6605
- C. 9.463
- D. 660.5

✓ Practice 4 · answer: B

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L}$$

given: 2500. mL · plan: mL → L → gal

$$2500. \cancel{\text{ mL}} \times \frac{10^{-3} \cancel{\text{ L}}}{1 \cancel{\text{ mL}}} \times \frac{1 \text{ gal}}{3.785 \cancel{\text{ L}}} = 0.6605 \text{ gal (answer B)}$$

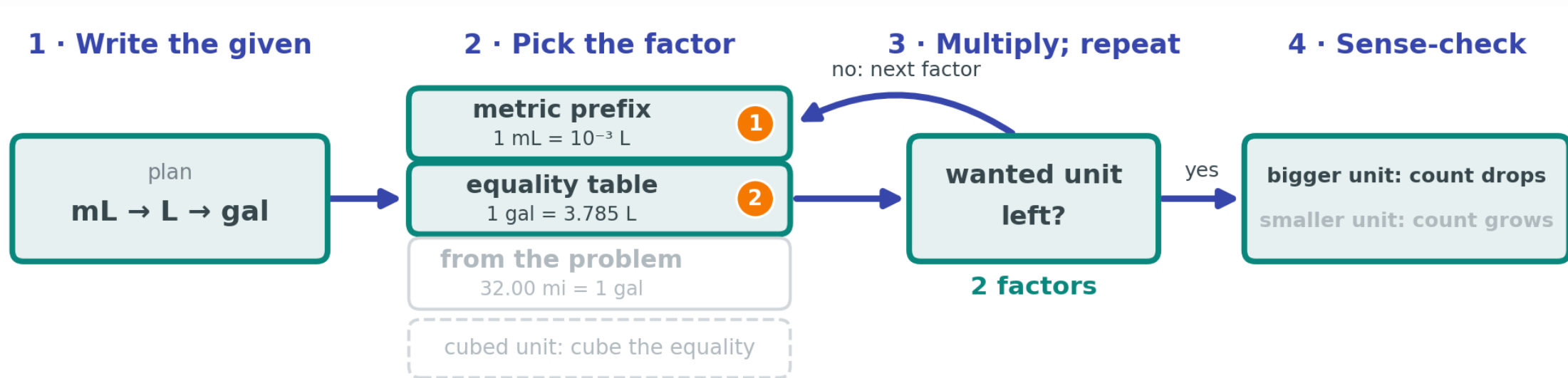
Four sig figs survive: 2500. carries four, and the English-metric factor 3.785 counts too, so nothing limits below four. A stopped mid-plan: 2500. mL is 2.500 L, and liters are not gallons. C flipped the gallon factor: $2500./1000 \times 3.785 = 9.463$. D treated milliliters as liters: $2500./3.785 = 660.5$.

 **Sense check:** A gallon is nearly four liters. The bottle holds 2.500 L, less than one gallon: 0.6605. ✓

✓ Practice 4: the route on the map

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L}$$

given: 2500. mL · found: 0.6605 gal



Sense check: A prefix factor to liters, then the gallon equality from the table. Two factors. A gallon is the bigger unit, so the count dropped. ✓

Worked example 4: a rate as a conversion factor

Step 1 • Write the given

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L} \cdot 0.62137 \text{ mi} = 1 \text{ km}$$

given: 1500. mL of gasoline • this car: 32.00 mi = 1 gal • wanted: km

A car gets **32.00 miles per gallon**. How many kilometers can it travel on **1500. mL** of gasoline?

Mileage is an equality for this car: 32.00 mi = 1 gal, a measured rating carrying four sig figs. The unit plan: mL → L → gal → mi → km.

✓ Worked example 4: solution

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L} \cdot 32.00 \text{ mi} = 1 \text{ gal} \cdot 0.62137 \text{ mi} = 1 \text{ km}$$

given: 1500. mL · plan: mL → L → gal → mi → km

Step 2 · Pick the factor that cancels its unit

$$1500. \cancel{\text{mL}} \times \frac{10^{-3} \text{ L}}{1 \cancel{\text{mL}}} = 1.500 \text{ L}$$

Step 3 · Multiply; repeat until the wanted unit survives

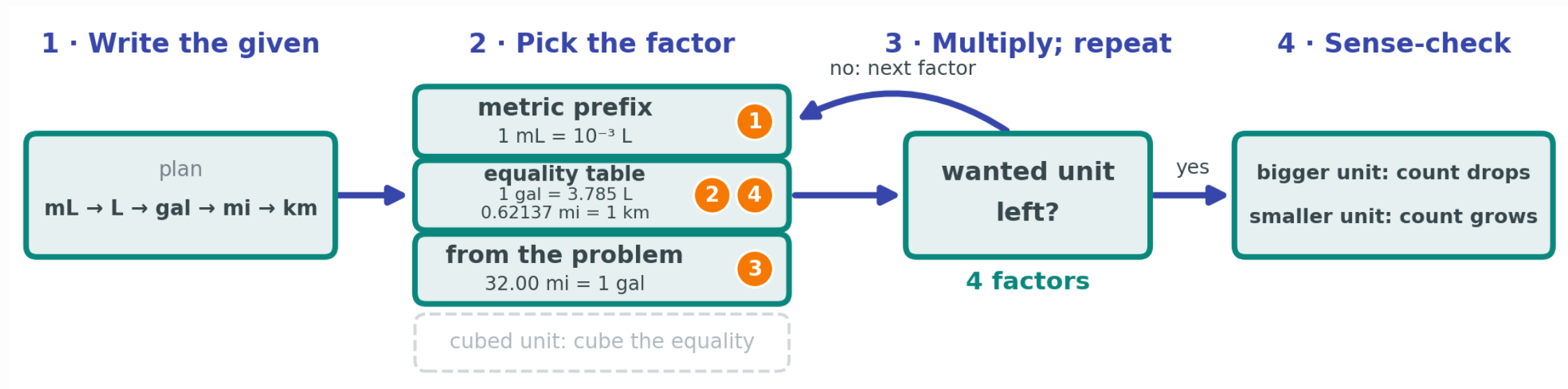
$$1500. \cancel{\text{mL}} \times \frac{10^{-3} \cancel{\text{L}}}{1 \cancel{\text{mL}}} \times \frac{1 \cancel{\text{gal}}}{3.785 \cancel{\text{L}}} \times \frac{32.00 \cancel{\text{mi}}}{1 \cancel{\text{gal}}} \times \frac{1 \text{ km}}{0.62137 \cancel{\text{mi}}} = 20.41 \text{ km}$$

 **Sense check:** 1500. mL is 1.5 L, under half a gallon: about 12.68 mi of driving. A kilometer is shorter than a mile, so the count in kilometers reads larger: 20.41. ✓

✓ Worked example 4: the route on the map

$$1 \text{ mL} = 10^{-3} \text{ L} \cdot 1 \text{ gal} = 3.785 \text{ L} \cdot 32.00 \text{ mi} = 1 \text{ gal} \cdot 0.62137 \text{ mi} = 1 \text{ km}$$

given: 1500. mL · found: 20.41 km



 **Sense check:** Four factors: a prefix, the gallon equality, the car's mileage, then the mile equality. Only the mileage comes from the problem; it holds for this car alone. ✓



Practice 5

$1 \text{ gal} = 3.785 \text{ L} \cdot \text{this hose: } 12.0 \text{ L} = 1 \text{ min} \cdot 1 \text{ min} = 60 \text{ s}$
given: 5.00 gal bucket · wanted: s to fill

A garden hose delivers **12.0 L per minute**. How many seconds does it take to fill a **5.00 gal** bucket?

- A. 6.60
- B. 25.0
- C. 1.58
- D. 94.6

✓ Practice 5 · answer: D

$$1 \text{ gal} = 3.785 \text{ L} \cdot 12.0 \text{ L} = 1 \text{ min} \cdot 1 \text{ min} = 60 \text{ s}$$

given: 5.00 gal · plan: gal → L → min → s

$$5.00 \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} \times \frac{1 \text{ min}}{12.0 \text{ L}} \times \frac{60 \text{ s}}{1 \text{ min}} = \mathbf{94.6 \text{ s (answer D)}}$$

The flow rate is a measured equality, 12.0 L = 1 min, and it converts liters to minutes. A flipped the gallon factor: 5.00 / 3.785 / 12.0 × 60 = 6.60. B skipped the gallon hop and treated gallons as liters: 5.00 / 12.0 × 60 = 25.0. C stopped at minutes: 5.00 × 3.785 / 12.0 = 1.58, a count of min, not s.

 **Sense check:** 5.00 gal is about 19 L. At 12 L each minute that takes a bit over a minute and a half: 94.6 s. ✓

✓ Practice 5: the route on the map

$$1 \text{ gal} = 3.785 \text{ L} \cdot 12.0 \text{ L} = 1 \text{ min} \cdot 1 \text{ min} = 60 \text{ s}$$

given: 5.00 gal · found: 94.6 s

1 · Write the given

plan
gal → L → min → s

2 · Pick the factor

metric prefix
1 mL = 10⁻³ L

equality table
1 gal = 3.785 L
1 min = 60 s

from the problem
12.0 L = 1 min

cubed unit: cube the equality

3 · Multiply; repeat

no: next factor

wanted unit
left?

3 factors

4 · Sense-check

bigger unit: count drops
smaller unit: count grows

 **Sense check:** Three factors: the gallon equality, the hose's flow rate from the problem, then 1 min = 60 s from the table. ✓



Practice 6

$$1 \text{ kg} = 2.205 \text{ lb}$$

given: 46.0 lb child · wanted: mL of liquid per dose

A child weighing **46.0 lb** is prescribed amoxicillin at **25.0 mg per kg** of body mass. The liquid medicine holds **250. mg per 5.00 mL**. How many milliliters is one dose?

- A. 522
- B. 23.0
- C. 2.61×10^4
- D. 50.7
- E. 10.4

✓ Practice 6 · answer: E

1 kg = 2.205 lb · this order: 25.0 mg = 1 kg · this liquid: 250. mg = 5.00 mL
given: 46.0 lb · plan: lb → kg → mg → mL

$$46.0 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} \times \frac{25.0 \text{ mg}}{1 \text{ kg}} \times \frac{5.00 \text{ mL}}{250. \text{ mg}} = 10.4 \text{ mL (answer E)}$$

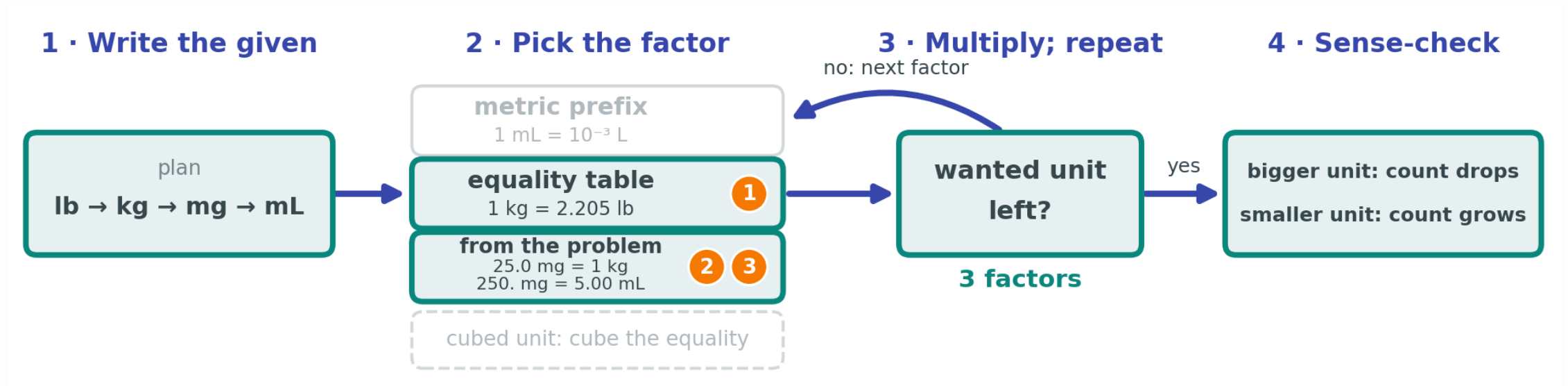
A stopped at milligrams: $46.0 / 2.205 \times 25.0 = 522 \text{ mg}$, a mass, not a volume. B treated pounds as kilograms: $46.0 \times 25.0 \times 5.00 / 250. = 23.0$. C flipped the liquid's factor: $522 \times 250. / 5.00 = 2.61 \times 10^4$. D flipped the pound factor: $46.0 \times 2.205 \times 25.0 \times 5.00 / 250. = 50.7$.

 **Sense check:** The child is about 20.9 kg, so the dose is about 522 mg. Each 5.00 mL carries 250. mg, so the dose is a little over two 5 mL spoonfuls: 10.4 mL. ✓

✓ Practice 6: the route on the map

$$1 \text{ kg} = 2.205 \text{ lb} \cdot 25.0 \text{ mg} = 1 \text{ kg} \cdot 250. \text{ mg} = 5.00 \text{ mL}$$

given: 46.0 lb · found: 10.4 mL

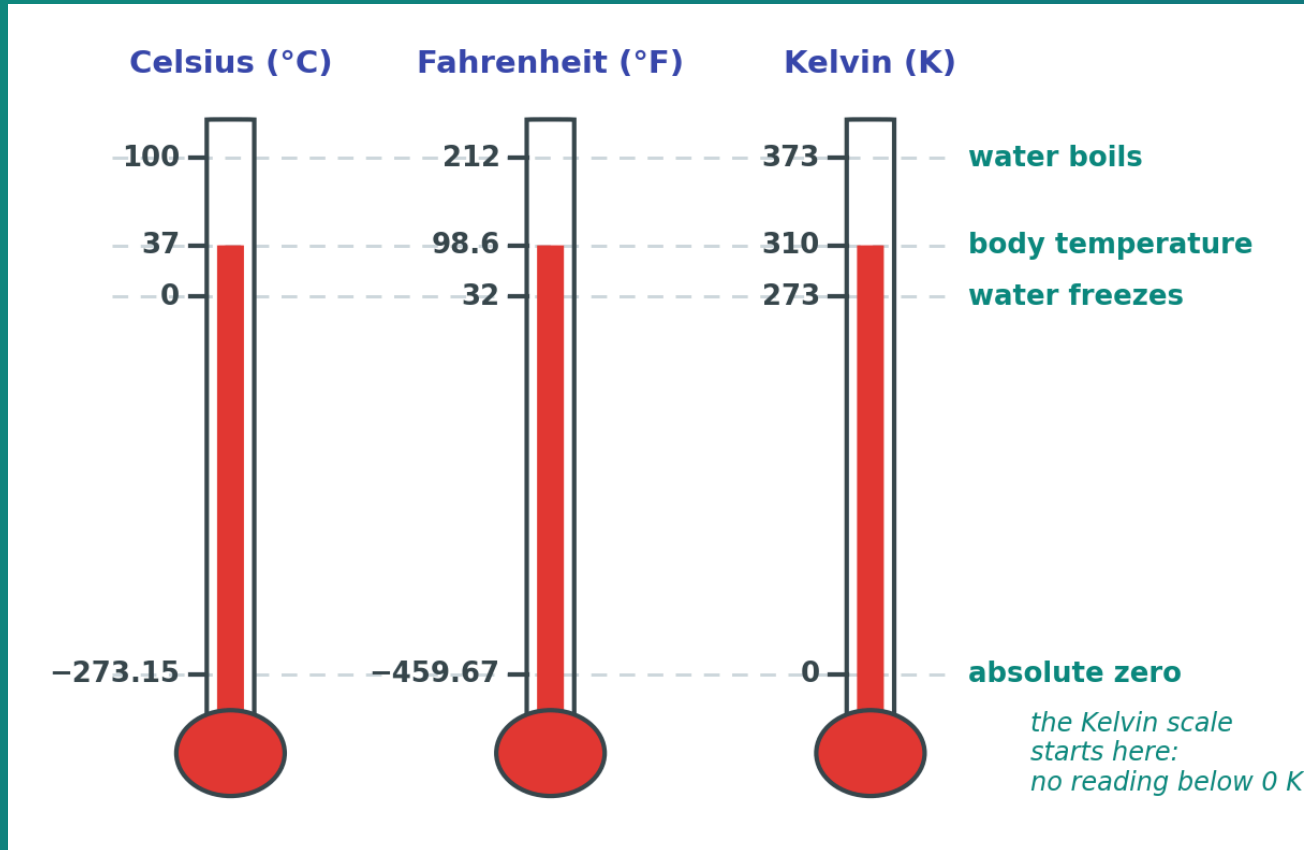


Sense check: Three factors: the pound equality from the table, then two from the problem, the dose per kilogram and the strength of the liquid. ✓

5 · Temperature Scales & Conversions

Convert a temperature reading among Celsius, Fahrenheit, and Kelvin, and handle a temperature difference correctly in any scale.

One temperature, three numbers



A Phoenix forecast says 95. A lab thermometer in the same air reads 35. A gas-law table lists the day as 308. Same afternoon, three scales.

Temperature: how hot or cold

water freezes: $32\text{ }^{\circ}\text{F} = 0\text{ }^{\circ}\text{C} = 273.15\text{ K}$

read on a thermometer · three scales, one physical temperature · SI unit: the kelvin (K)

Temperature measures how hot or cold a substance is. Each scale gives the same temperature its own number.

Which is colder: $0\text{ }^{\circ}\text{C}$ or $0\text{ }^{\circ}\text{F}$?

$0\text{ }^{\circ}\text{C}$ is water's freezing point · $0\text{ }^{\circ}\text{F}$ sits $32\text{ }^{\circ}\text{F}$ below it: $32\text{ }^{\circ}\text{F} - 32\text{ }^{\circ}\text{F} = 0\text{ }^{\circ}\text{F}$

$0\text{ }^{\circ}\text{F}$ is colder ✓ · a zero on two scales marks two different temperatures

A scale is two choices

water freezes: $0\text{ }^{\circ}\text{C} = 32\text{ }^{\circ}\text{F} = 273.15\text{ K}$

water boils: $100\text{ }^{\circ}\text{C} = 212\text{ }^{\circ}\text{F} = 373.15\text{ K}$

A temperature scale fixes two things: where its zero sits and how large one degree is. Celsius, Fahrenheit, and Kelvin mark the same physical events with different numbers.

Why Kelvin exists

$$0 \text{ K} = -273.15 \text{ }^{\circ}\text{C}$$

absolute zero: the coldest possible temperature · no negative kelvins

Kelvin puts its zero at absolute zero, where molecular motion reaches its minimum. Every Kelvin reading is positive. Gas volumes and pressures are proportional to Kelvin temperature, so the gas laws require this scale.

Celsius to Kelvin: shift the zero

$$K = ^\circ\text{C} + 273.15 \quad ^\circ\text{C} = K - 273.15$$

$$25\text{ }^\circ\text{C} + 273.15 = 298.15\text{ K} \quad 298.15\text{ K} - 273.15 = 25\text{ }^\circ\text{C}$$

A kelvin and a Celsius degree are the same size. Converting a reading is a single shift: add 273.15 going to Kelvin, subtract it coming back.

Celsius to Fahrenheit: stretch, then shift

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32 \quad \cdot \quad ^{\circ}\text{C} = (^{\circ}\text{F} - 32) / 1.8$$

$$\text{freezing to boiling: } 212 - 32 = 180 \text{ F}^{\circ} \text{ across } 100 \text{ C}^{\circ} \cdot 180/100 = 1.8$$

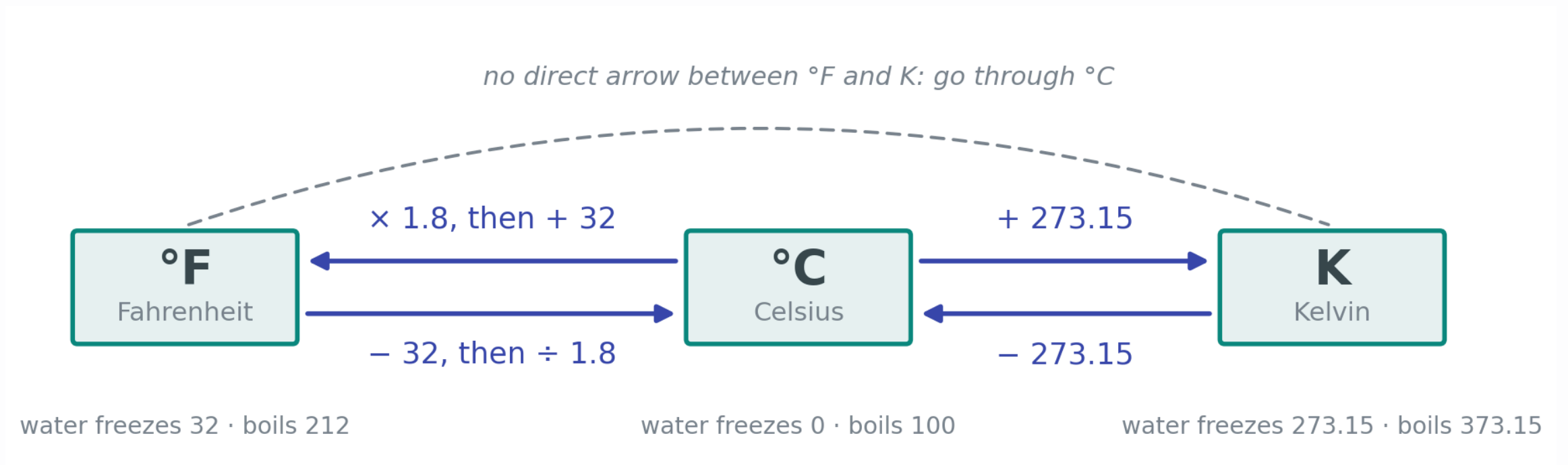
Fahrenheit degrees are smaller, so the reading is stretched by 1.8 before the zero shifts by 32. Going back reverses the order: subtract 32 first, then divide by 1.8.



The method

1. **Name the scales:** given and wanted.
2. **Pick the linking equation.**
3. **Solve, one operation at a time:** between $^{\circ}\text{F}$ and $^{\circ}\text{C}$, the 32 shifts last going in, first coming back.
4. **Sense-check** against the reference points.

One map for every temperature problem



Every route runs through Celsius. Kelvin is one shift away. Fahrenheit is a stretch and a shift away, in a fixed order. No arrow joins Fahrenheit to Kelvin directly.

Guided example: a weather balloon

Step 1 • Name the scales

$$^{\circ}\text{C} = \text{K} - 273.15$$

given: 241 K • wanted: $^{\circ}\text{C}$

A weather balloon's sensor reads **241 K** high in the atmosphere. Express the reading in $^{\circ}\text{C}$.

On the map, find the given scale and the wanted one. Kelvin to Celsius is a single arrow.

✓ Guided example: solution

$$^{\circ}\text{C} = \text{K} - 273.15$$

given: 241 K · wanted: $^{\circ}\text{C}$

Step 2 · Pick the linking equation

Step 3 · Solve, one operation at a time

$$^{\circ}\text{C} = 241 - 273.15 = -32.15 \rightarrow -32\text{ }^{\circ}\text{C}$$

one subtraction · reported to the ones place of 241

Step 4 · Sense-check

 **Sense check:** 241 K sits below water's freezing point, 273.15 K, so the Celsius reading must be negative: $-32\text{ }^{\circ}\text{C}$.

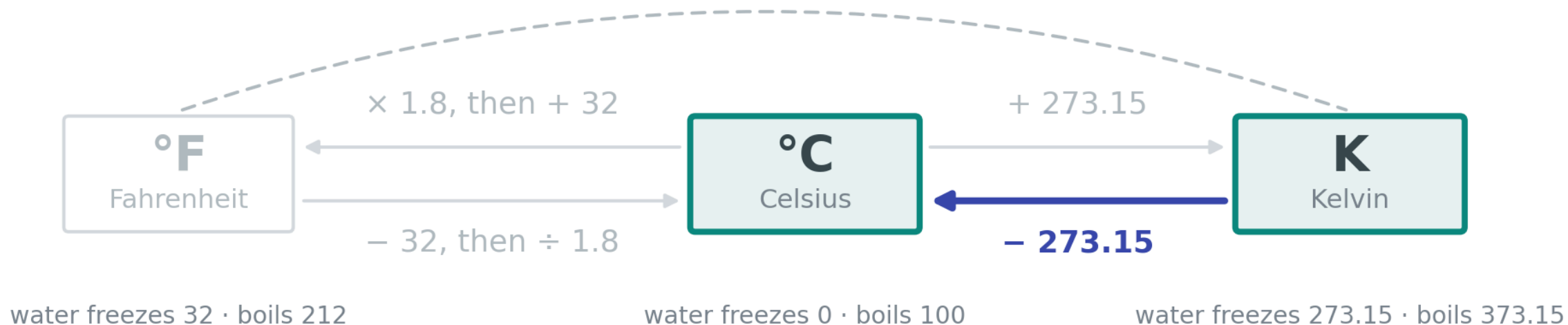
✓

✓ Guided example: the route on the map

$$^{\circ}\text{C} = \text{K} - 273.15$$

given: 241 K · found: -32°C

no direct arrow between $^{\circ}\text{F}$ and K: go through $^{\circ}\text{C}$



🧠 **Sense check:** One arrow, Kelvin to Celsius: subtract 273.15. No 1.8 and no 32 on this route. ✓



Practice 1

$$^{\circ}\text{C} = \text{K} - 273.15$$

given: 358 K · wanted: $^{\circ}\text{C}$

A sauna's air sensor logs **358 K**. What is this temperature in $^{\circ}\text{C}$?

- A. 631
- B. 85
- C. 185
- D. 153

✓ Practice 1 · answer: B

$$^{\circ}\text{C} = 358 - 273.15 = 84.85 \rightarrow 85 ^{\circ}\text{C} \text{ (answer B)}$$

one subtraction · reported to the ones place of 358

A added the shift instead of taking it off: $358 + 273.15 = 631.15$. C answered in Fahrenheit, the wrong scale: $1.8(84.85) + 32 = 184.73$. D stretched by 1.8 on the way to Celsius: $1.8 \times 84.85 = 152.73$; the 1.8 belongs to Fahrenheit.

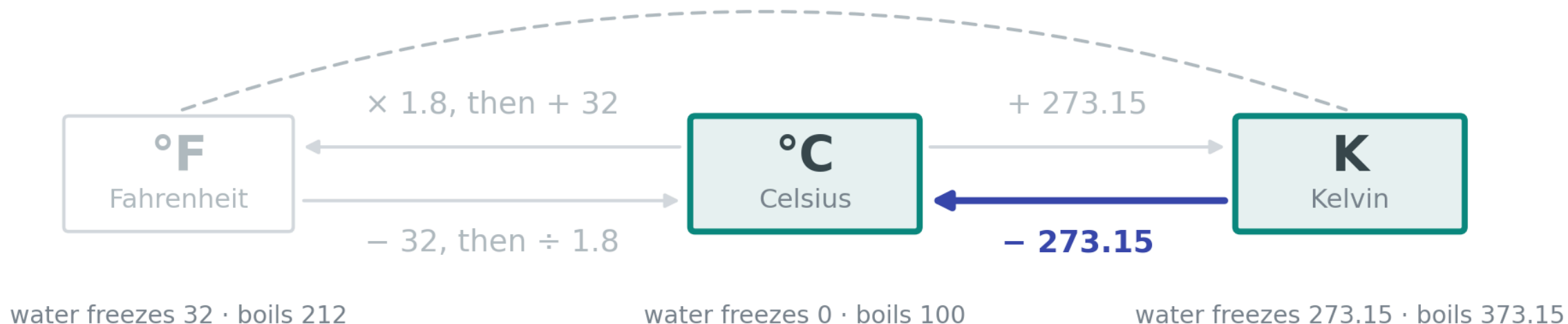
 **Sense check:** 358 K sits between water's freezing point (273.15 K) and its boiling point (373.15 K): 85 °C, hot but short of boiling. ✓

✓ Practice 1: the route on the map

$$^{\circ}\text{C} = \text{K} - 273.15$$

given: 358 K · found: 85 $^{\circ}\text{C}$

no direct arrow between $^{\circ}\text{F}$ and K: go through $^{\circ}\text{C}$



🧠 **Sense check:** One arrow, Kelvin to Celsius: subtract 273.15. A reading above 273.15 K lands above 0 $^{\circ}\text{C}$. ✓



Worked example 1

Step 1 • Name the scales

$$K = ^\circ C + 273.15$$

given: 37.0 °C • wanted: K

A water bath holds a protein sample at **37.0 °C**, body temperature. The instrument log records temperatures in kelvins. Express the reading in K.

✓ Worked example 1: solution

$$K = ^\circ\text{C} + 273.15$$

given: 37.0 °C · wanted: K

Step 2 · Pick the linking equation

Step 3 · Solve, one operation at a time

$$K = 37.0 + 273.15 = 310.15 \rightarrow 310.2 \text{ K}$$

one addition · reported to the tenths place of 37.0

Step 4 · Sense-check

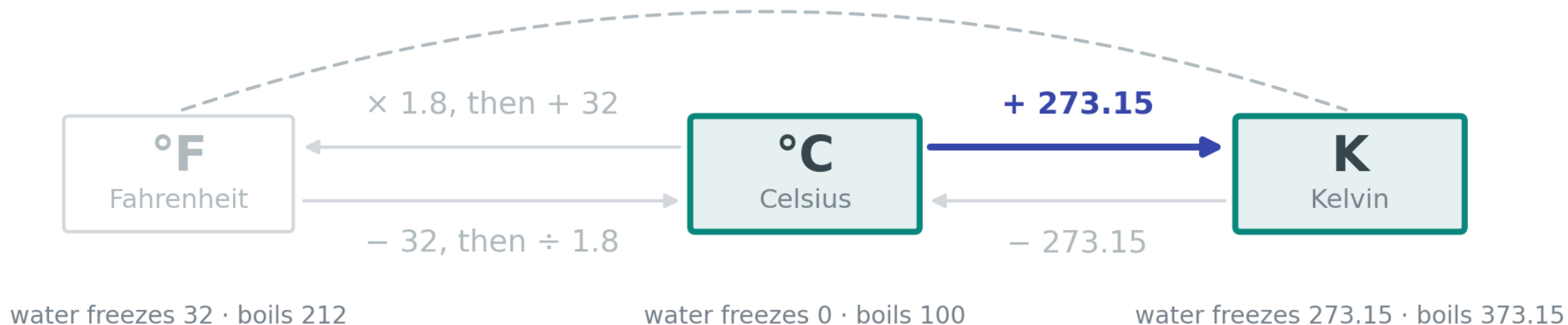
 **Sense check:** Body temperature sits between freezing (273.15 K) and boiling (373.15 K), nearer the freezing end: 310.2 K. ✓ The shift moved the number, not the warmth.

✓ Worked example 1: the route on the map

$$K = ^\circ C + 273.15$$

given: $37.0\ ^\circ C$ · found: $310.2\ K$

no direct arrow between $^\circ F$ and K : go through $^\circ C$



 **Sense check:** One arrow, Celsius to Kelvin: add 273.15. The degrees are the same size, so nothing stretches. ✓

Worked example 2

Step 1 • Name the scales

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32$$

given: 35.0 $^{\circ}\text{C}$ • wanted: $^{\circ}\text{F}$

The Phoenix thermometer reads **35.0 $^{\circ}\text{C}$** , and the forecast graphic wants Fahrenheit.

A common first attempt shifts before it stretches: add the 32 first. Test it.

✓ Worked example 2: solution

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32$$

given: 35.0°C · wanted: $^{\circ}\text{F}$

A common first attempt

$$(35.0 + 32) \times 1.8 = 120.6^{\circ}\text{F} \times$$

the 32 got stretched along with the reading

Step 2 · Pick the linking equation

Step 3 · Solve, one operation at a time

$$^{\circ}\text{F} = 1.8(35.0) + 32 = 63.0 + 32 = 95.0^{\circ}\text{F}$$

stretch: $1.8 \times 35.0 = 63.0$ · shift: $63.0 + 32 = 95.0$ ✓

Step 4 · Sense-check

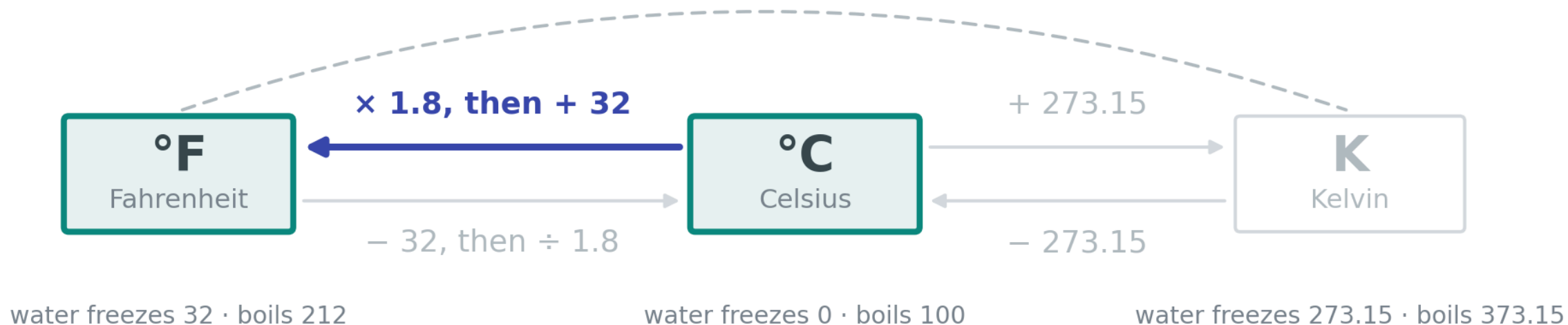
🧠 **Sense check:** 35.0°C is just under body temperature (37°C), so $^{\circ}\text{F}$ must land just under 98.6: 95.0. ✓

✓ Worked example 2: the route on the map

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32$$

given: 35.0°C · found: 95.0°F

no direct arrow between $^{\circ}\text{F}$ and K : go through $^{\circ}\text{C}$



🧠 **Sense check:** One arrow, two operations in the arrow's order: $\times 1.8$ first, then $+ 32$. ✓

Your turn: Fahrenheit to Celsius

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32) / 1.8$$

given: 68 $^{\circ}\text{F}$ · wanted: $^{\circ}\text{C}$

A thermostat is set to **68 $^{\circ}\text{F}$** . Coming back to Celsius, the order reverses: the 32 comes off first, then the 1.8 divides out.

$$^{\circ}\text{C} = (68 - 32) / 1.8 = \underline{\hspace{1cm}} / 1.8 = \underline{\hspace{1cm}} ^{\circ}\text{C}$$

Fill both blanks.

$$^{\circ}\text{C} = (68 - 32) / 1.8 = 36 / 1.8 = 20. ^{\circ}\text{C}$$

subtract first · divide last · a 20 $^{\circ}\text{C}$ room

A difference is not a reading

rise from 20 °C to 45 °C: $\Delta T = 45 - 20 = 25 \text{ C}^\circ = 25 \text{ K}$

the 273.15 shift cancels in the subtraction · in Fahrenheit: $1.8 \times 25 = 45 \text{ F}^\circ$

A reading marks a point; a difference measures a gap. Shifting both endpoints by 273.15 leaves the gap unchanged: a Celsius difference is a Kelvin difference, degree for degree.

⚠ Where this goes wrong

Adding 273.15 to a Fahrenheit reading. $98.6\text{ }^{\circ}\text{F} + 273.15 = 371.75$, a number on no scale. The 273.15 shift belongs to Celsius only. Route through Celsius: $(98.6 - 32) / 1.8 = 37.0\text{ }^{\circ}\text{C}$, then $37.0 + 273.15 = 310.15\text{ K}$.

Stretching by 1.8 on the way to Kelvin. $1.8(25) + 273.15 = 318.15$ is wrong; the 1.8 belongs to Fahrenheit. Celsius and Kelvin degrees are the same size: $25 + 273.15 = 298.15\text{ K}$.

Converting a temperature difference like a reading. A rise of $25\text{ }^{\circ}\text{C}$ is not a rise of 298.15 K . The shift cancels between the two endpoints: a $25\text{ }^{\circ}\text{C}$ rise is a 25 K rise, and $1.8 \times 25 = 45\text{ }^{\circ}\text{F}$.

 Practice 2

$$K = ^\circ\text{C} + 273.15$$

given: $-78\text{ }^\circ\text{C}$ · wanted: K

Dry ice sublimates at $-78\text{ }^\circ\text{C}$. What is this temperature in kelvins?

- A. 351
- B. -108
- C. -351
- D. 195

✓ Practice 2 · answer: D

$$K = -78 + 273.15 = 195.15 \rightarrow 195 \text{ K (answer D)}$$

–78 is known to the ones place · report 195 K

A dropped the minus sign: $78 + 273.15 = 351.15$. C subtracted the shift instead of adding it: $-78 - 273.15 = -351.15$, and negative kelvins do not exist. B built Fahrenheit instead of Kelvin: $1.8(-78) + 32 = -108.4$.

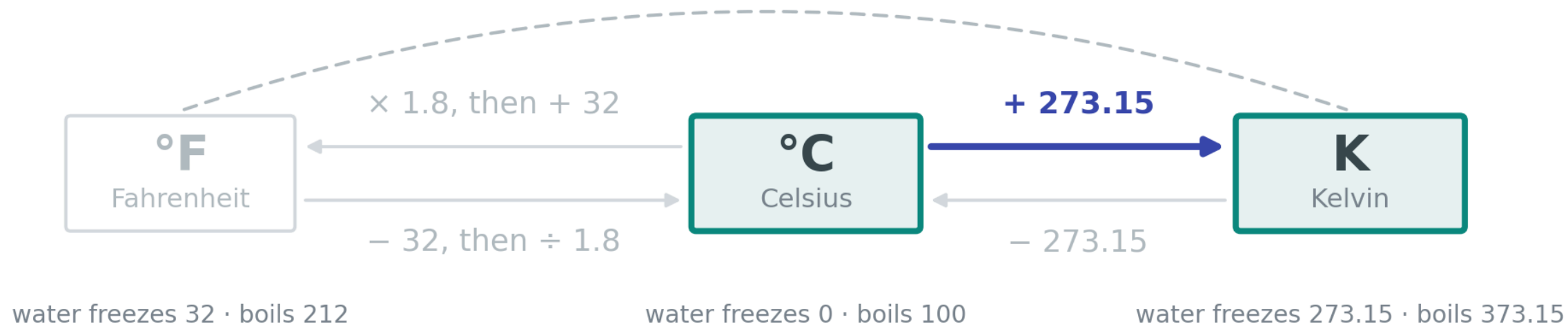
🧠 **Sense check:** Dry ice is cold, but absolute zero is far colder: 195 K sits 78 degrees below freezing (273.15 K) and well above 0 K. Any negative Kelvin answer fails before the arithmetic starts. ✓

✓ Practice 2: the route on the map

$$K = ^\circ C + 273.15$$

given: $-78\ ^\circ C$ · found: 195 K

no direct arrow between $^\circ F$ and K: go through $^\circ C$



🧠 **Sense check:** One arrow, Celsius to Kelvin: add 273.15 to the signed reading, $-78 + 273.15$. ✓



Practice 3

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32 \cdot \text{K} = ^{\circ}\text{C} + 273.15$$

given: 223 K · wanted: $^{\circ}\text{F}$

A cold-test chamber holds a battery pack at **223 K**. Its door display reads in $^{\circ}\text{F}$. What should the display show?

- A. -58
- B. -50
- C. 4
- D. 122
- E. 433

✓ Practice 3 · answer: A

$^{\circ}\text{C} = 223 - 273.15 = -50.15 \rightarrow ^{\circ}\text{F} = 1.8(-50.15) + 32 = -58.27 \rightarrow -58 ^{\circ}\text{F}$ (answer A)
no direct K $\rightarrow$ $^{\circ}\text{F}$ equation: route through Celsius · shift, then stretch, then shift · ones place from 223

B stopped at Celsius: -50.15 , never stretched or shifted into $^{\circ}\text{F}$. C divided by 1.8, the coming-back move: $-50.15 / 1.8 + 32 = 4.14$. D dropped the minus sign: $1.8(50.15) + 32 = 122.27$. E fed kelvins straight into the Fahrenheit equation: $1.8(223) + 32 = 433.4$.

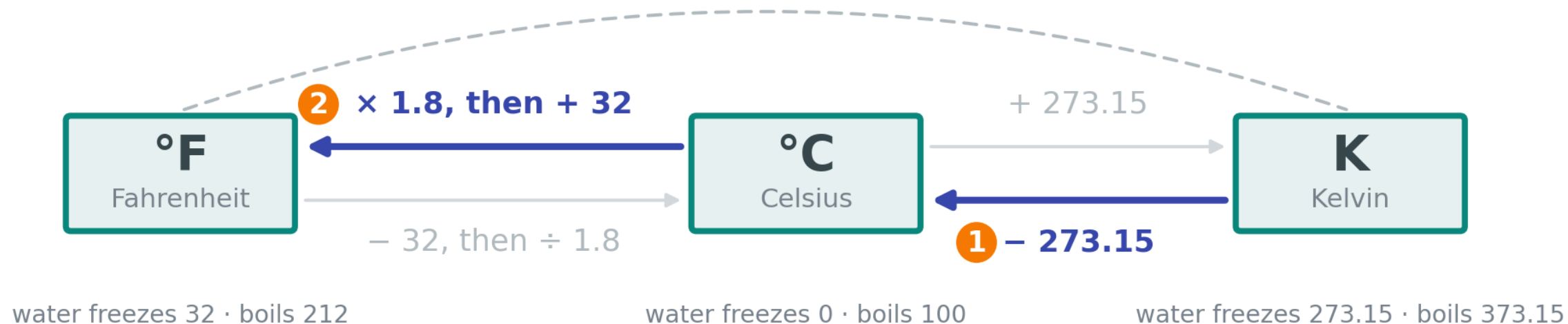
 **Sense check:** Below -40 the Fahrenheit number is the more negative one: $-50 ^{\circ}\text{C}$ lands at $-58 ^{\circ}\text{F}$. Far colder than a kitchen freezer, far above 0 K. ✓

✓ Practice 3: the route on the map

$$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32 \cdot \text{K} = ^{\circ}\text{C} + 273.15$$

given: 223 K · found: –58 °F

no direct arrow between °F and K: go through °C



🧠 **Sense check:** No arrow joins Kelvin to Fahrenheit. Move 1: – 273.15. Move 2: × 1.8, then + 32. ✓

 Practice 4

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32) / 1.8 \cdot \text{K} = ^{\circ}\text{C} + 273.15$$

given: 250. $^{\circ}\text{F}$ · wanted: K

An autoclave sterilizes lab glassware at **250. $^{\circ}\text{F}$** . What is this temperature in kelvins?

- A. 491
- B. 121
- C. 394
- D. 523
- E. 380

✓ Practice 4 · answer: C

Two moves are needed: Fahrenheit to Celsius, then Celsius to Kelvin.

$$^{\circ}\text{C} = (250. - 32) / 1.8 = 121.11 \cdot \text{K} = 121.11 + 273.15 = 394.26 \rightarrow 394 \text{ K (answer C)}$$

given: 250. °F · subtract 32 first, divide by 1.8 second · ones place from 250.

A never divided by 1.8: $250. - 32 + 273.15 = 491.15$. B stopped at Celsius: $(250. - 32) / 1.8 = 121.11$. D added 273.15 straight to the Fahrenheit reading: $250. + 273.15 = 523.15$. E divided before subtracting: $250. / 1.8 - 32 + 273.15 = 380.04$.

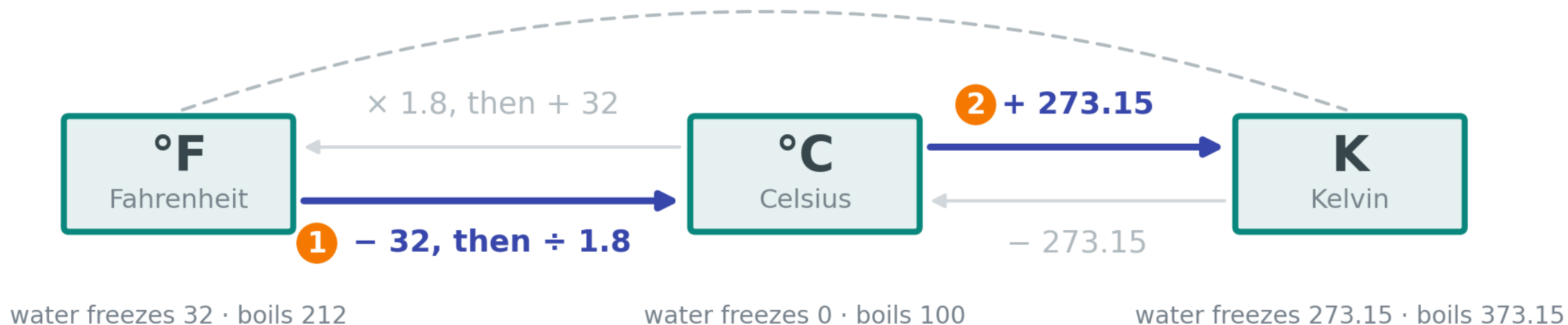
 **Sense check:** 250. °F tops water's boiling point, 212 °F, so the answer must top 373.15 K: 394 K, about 21 degrees above boiling. ✓

✓ Practice 4: the route on the map

$$^{\circ}\text{C} = (^{\circ}\text{F} - 32) / 1.8 \cdot \text{K} = ^{\circ}\text{C} + 273.15$$

given: 250. $^{\circ}\text{F}$ · found: 394 K

no direct arrow between $^{\circ}\text{F}$ and K: go through $^{\circ}\text{C}$



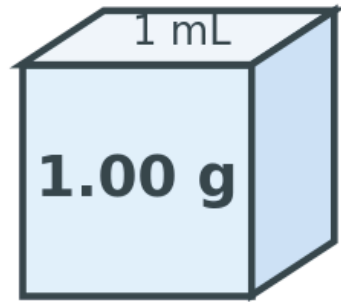
🧠 **Sense check:** No arrow joins Fahrenheit to Kelvin. Move 1: $- 32$, then $\div 1.8$. Move 2: $+ 273.15$. ✓

6 · Density as a Conversion Factor

Use a density to convert between the mass and the volume of a material, alone or chained with other conversion factors on one rail.

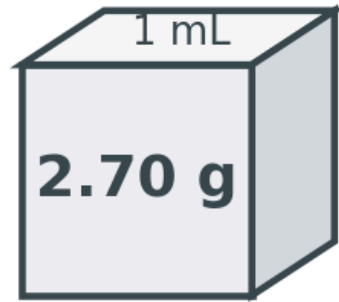
Same volume, different mass

the same milliliter, three different masses



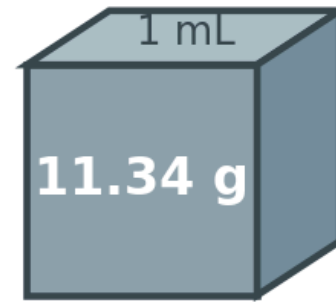
water

1.00 g/mL



aluminum

2.70 g/mL



lead

11.34 g/mL

Three cubes, the same 1 mL of space. Water: 1.00 g. Aluminum: 2.70 g. Lead: 11.34 g. Each material packs its own mass into a milliliter.

Density: mass per milliliter

ethanol 0.789 · water 1.00 · aluminum 2.70 · iron 7.87 · silver 10.5 · gold 19.3

density in g/mL: the mass, in grams, that fills 1 mL of the material

Each material packs a fixed mass into each milliliter. That rate, in grams per milliliter, is its **density**, a physical property. A measured density identifies the material.

Density = mass ÷ volume

$$\text{density} = \text{mass} \div \text{volume}$$

solids and liquids: g/mL or g/cm³, with 1 cm³ = 1 mL · gases: g/L

Divide a mass by the volume it fills. The units divide too: grams over milliliters gives g/mL.

A 2.00 cm³ cube of aluminum has a mass of 5.40 g. Find its density.

$$d = \frac{5.40 \text{ g}}{2.00 \text{ cm}^3} = \mathbf{2.70 \text{ g/cm}^3}, \text{ the same as } 2.70 \text{ g/mL}$$

A density is an equality

19.3 g of gold = 1 mL of gold
density of gold: 19.3 g/mL

Every equality gives a conversion factor. This one converts between mass and volume:

$$\frac{19.3 \text{ g Au}}{1 \text{ mL Au}} \text{ or } \frac{1 \text{ mL Au}}{19.3 \text{ g Au}}$$

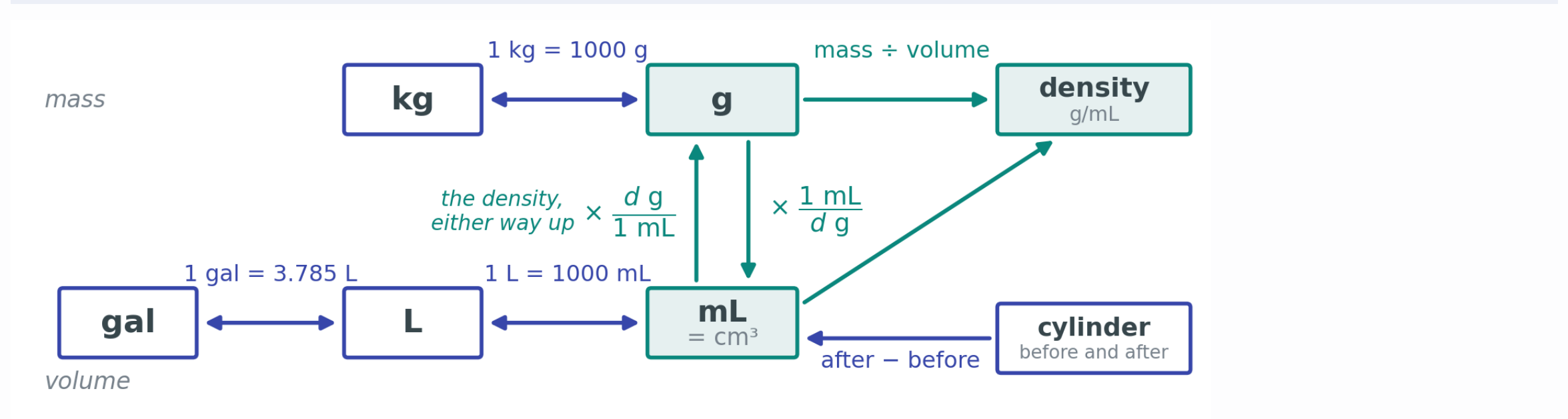
Write it so the **given unit cancels**.



The method

1. **Map the route:** given unit $\rightarrow$ desired unit.
2. **Start with the given.**
3. **Write the density fraction** so the unit to cancel sits in the denominator.
4. **Multiply and check** that only the desired unit survives.

One map for every density problem



Metric factors move along a row. Only the density crosses between mass and volume, written either way up. Mass $\div$ volume builds a density from data.



Guided example: glycerol by the milliliter

Step 1 • Map the route

1.26 g glycerol = 1 mL glycerol
given: 500. mL • wanted: g • route: mL → g

A soap maker pours **500. mL of glycerol** into a mixing tank. What mass of glycerol goes in?

Every milliliter of glycerol has a mass of 1.26 g.

✓ Guided example: solution

1.26 g glycerol = 1 mL glycerol
given: 500. mL · wanted: g · route: mL → g

Step 2 · Start with the given

Step 3 · Write the density fraction

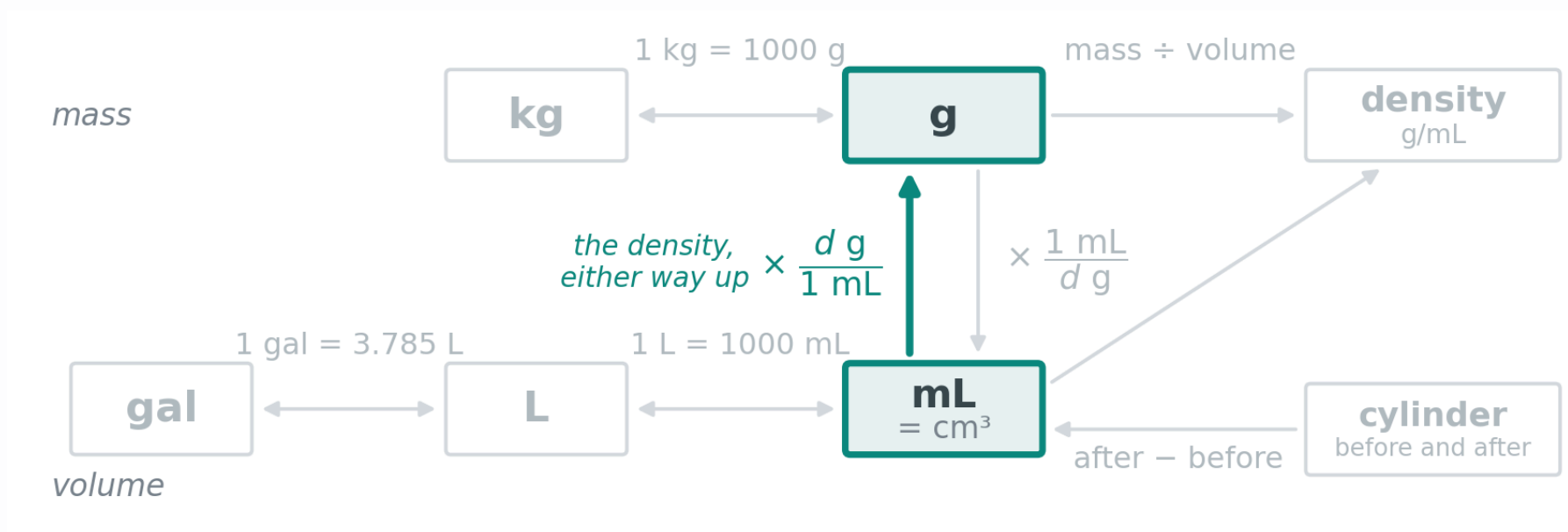
Step 4 · Multiply and check

$$500. \cancel{\text{ mL glycerol}} \times \frac{1.26 \text{ g glycerol}}{1 \cancel{\text{ mL glycerol}}} = 630. \text{ g glycerol}$$

🧠 **Sense check:** The same tank of water would take 500. g. Glycerol packs a little more into each milliliter: 630. g ✓

✓ Guided example: the route on the map

1.26 g glycerol = 1 mL glycerol
given: 500. mL · found: 630. g



🧠 **Sense check:** One arrow, mL → g: the density with milliliters on the bottom. ✓



Practice 1

$$7.85 \text{ g steel} = 1 \text{ cm}^3 \text{ steel}$$

density of steel: 7.85 g/cm^3

A steel ball bearing has a volume of 4.20 cm^3 . What is its mass, in grams?

- A. 0.535
- B. 3.30
- C. 33.0
- D. 1.87

✓ Practice 1 · answer: C

$$7.85 \text{ g steel} = 1 \text{ cm}^3 \text{ steel}$$

given: 4.20 cm^3 · wanted: g · route: $\text{cm}^3 \rightarrow \text{g}$

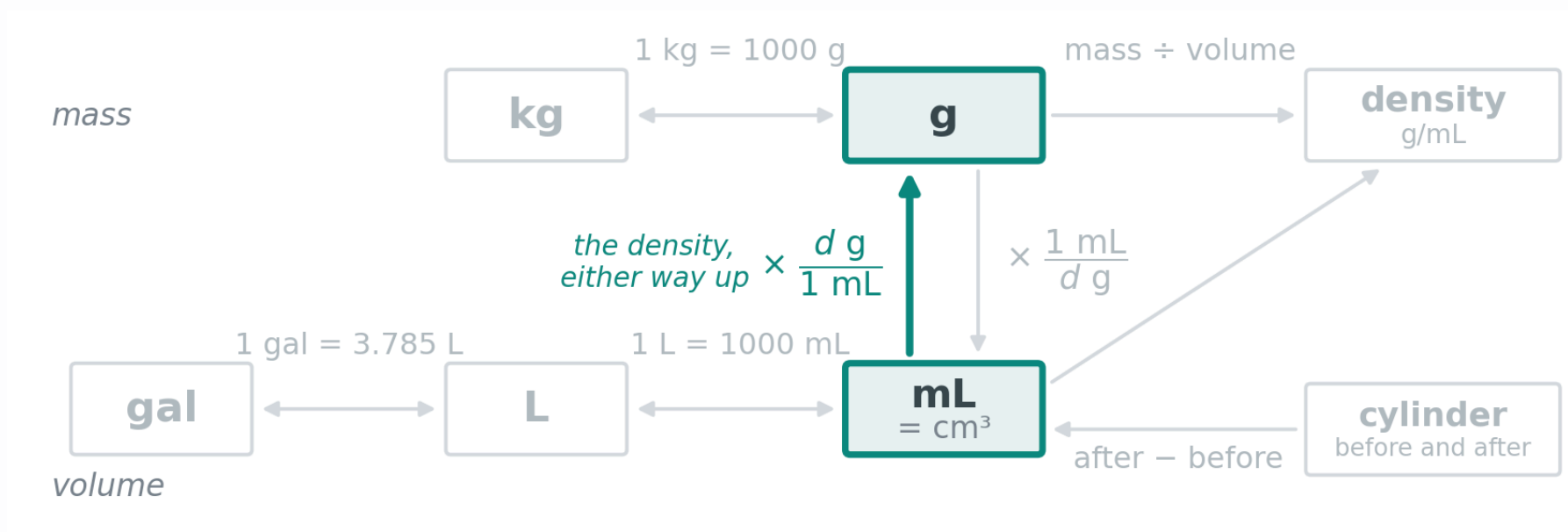
$$4.20 \text{ cm}^3 \text{ steel} \times \frac{7.85 \text{ g steel}}{1 \text{ cm}^3 \text{ steel}} = 33.0 \text{ g steel (answer C)}$$

A flipped the fraction: $4.20 \div 7.85 = 0.535$, and nothing cancels. B slipped a decimal: $4.20 \times 7.85 = 33.0$, not 3.30. D divided the density by the volume: $7.85 \div 4.20 = 1.87$, not a mass.

🧠 **Sense check:** Estimate: about 8 g in each of about 4 cm^3 is near 32 g, so 33.0 g fits. ✓

✓ Practice 1: the route on the map

7.85 g steel = 1 cm³ steel
given: 4.20 cm³ · found: 33.0 g



🧠 **Sense check:** The path of the guided example. A cm³ is a mL, so the same arrow carries cm³ to grams. ✓



Worked example 1: volume from mass

Step 1 • Map the route

$$19.3 \text{ g Au} = 1 \text{ mL Au}$$

given: 25.0 g · wanted: mL · route: g → mL

A pendant contains **25.0 g of gold**. What volume of gold is that?

A common first attempt: multiply by the density fraction as written, 19.3 g over 1 mL. Test the units.

✓ Worked example 1: solution

19.3 g Au = 1 mL Au
given: 25.0 g · wanted: mL · route: g → mL

A common first attempt

$$25.0 \text{ g Au} \times \frac{19.3 \text{ g Au}}{1 \text{ mL Au}} = 483 \text{ g}^2/\text{mL} \times$$

Step 2 · Start with the given

Step 3 · Write the density fraction

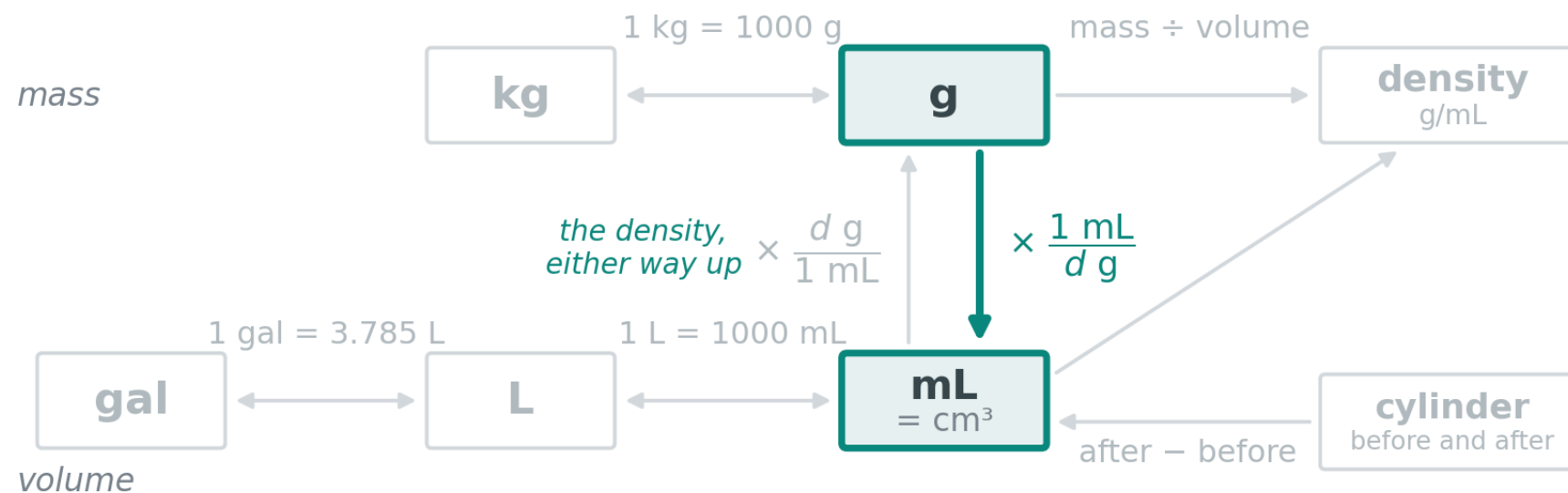
Step 4 · Multiply and check

$$25.0 \text{ g Au} \times \frac{1 \text{ mL Au}}{19.3 \text{ g Au}} = 1.30 \text{ mL Au}$$

🧠 **Sense check:** Gold packs 19.3 g into each milliliter, so 25.0 g fits in barely more than one: 1.30 mL. ✓

✓ Worked example 1: the route on the map

19.3 g Au = 1 mL Au
given: 25.0 g · found: 1.30 mL



🧠 **Sense check:** One arrow, g → mL: the density flipped, grams on the bottom. The given unit picks the orientation.

✓

Your turn: ethanol

0.789 g ethanol = 1 mL ethanol

given: 50.0 g · wanted: mL

A hand-sanitizer recipe calls for **50.0 g of ethanol**, measured out by volume.

$$50.0 \text{ g} \times \frac{\text{_____ mL}}{\text{_____ g}} = \text{_____ mL}$$

Fill the fraction so grams cancel, then compute.

$$50.0 \text{ g} \times \frac{1 \text{ mL}}{0.789 \text{ g}} = \mathbf{63.4 \text{ mL}}$$

Where this goes wrong

Flipping the density fraction. $25.0 \text{ g} \times (19.3 \text{ g} / 1 \text{ mL}) = 483 \text{ g}^2/\text{mL}$. Nothing cancels, and no quantity carries g^2/mL . Put the unit to cancel in the denominator: $25.0 \text{ g} \times (1 \text{ mL} / 19.3 \text{ g}) = 1.30 \text{ mL}$.

Rearranging $d = m/V$ from memory. A misremembered rearrangement gives $1.26 \div 500. = 0.00252 \text{ g/mL}^2$, which is not a mass. No rearranging is needed: start with the given and multiply by the fraction that cancels its unit.

A slipped decimal. A correct setup can still be keyed in wrong: 63.0 g instead of 630. g. Estimate first. 500 mL at about 1.3 g per milliliter is near 650 g, so 63.0 g cannot be right.



Practice 2

$10.5 \text{ g Ag} = 1 \text{ mL Ag}$
density of silver: 10.5 g/mL

A silversmith buys a **170. g silver ingot**. What volume, in mL, does the ingot occupy?

- A. 0.0618
- B. 16.2
- C. 1.62
- D. 1.79×10^3

✓ Practice 2 · answer: B

$$10.5 \text{ g Ag} = 1 \text{ mL Ag}$$

given: 170. g · wanted: mL · route: g → mL

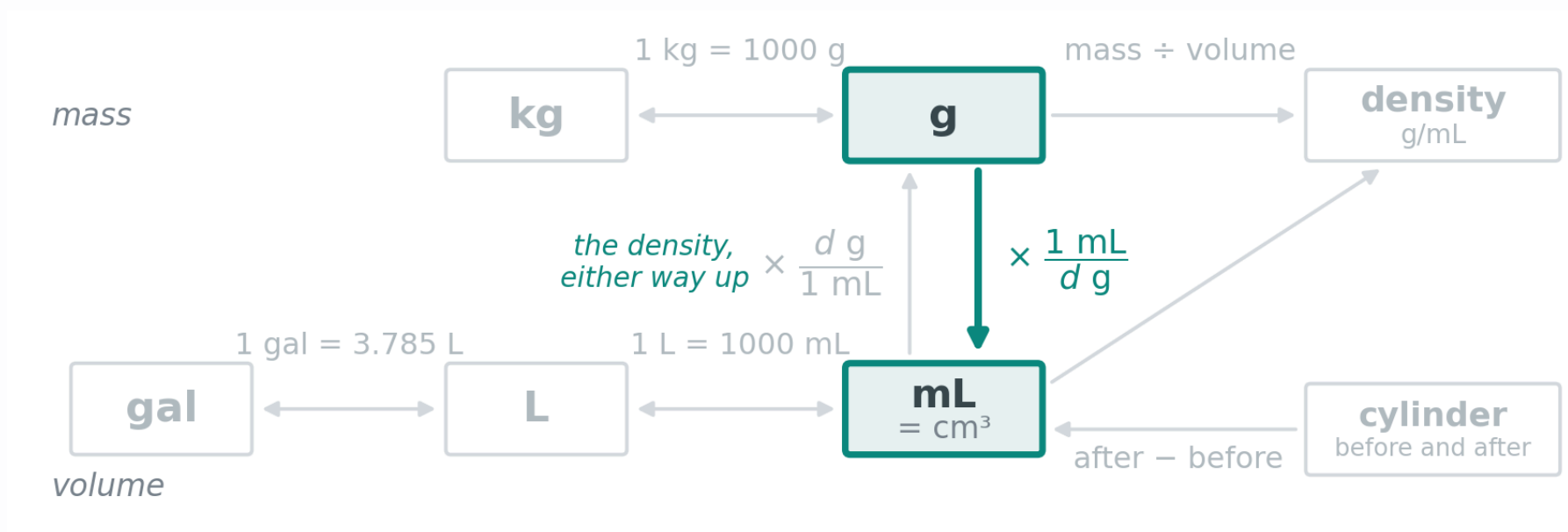
$$170. \text{ g Ag} \times \frac{1 \text{ mL Ag}}{10.5 \text{ g Ag}} = 16.2 \text{ mL Ag (answer B)}$$

A divided the density by the mass: $10.5 \div 170. = 0.0618$, and its units are not milliliters. C slipped a decimal: $170. \div 10.5 = 16.2$, not 1.62. D flipped the fraction: $170. \times 10.5 = 1.79 \times 10^3$, with units of g^2/mL .

 **Sense check:** Silver packs 10.5 g into each milliliter, so 170. g occupies far fewer milliliters than its grams: 16.2. ✓

✓ Practice 2: the route on the map

10.5 g Ag = 1 mL Ag
given: 170. g · found: 16.2 mL



🧠 **Sense check:** The path of worked example 1: one arrow, g → mL, grams on the bottom of the density. ✓



Worked example 2: an unknown metal

417 g of metal pellets · submerged, they raise the water level by 53.0 mL
given: 417 g and 53.0 mL · wanted: density, in g/mL

Candidate densities, in g/mL:

aluminum	iron	copper	silver	lead
2.70	7.87	8.96	10.5	11.34

A bin of unlabeled gray pellets arrives at a recycling yard. Density identifies the metal.

Build the density from the data, units in place, and match it to the table.

✓ Worked example 2: solution

417 g of metal pellets · 53.0 mL of water displaced
wanted: density, in g/mL

Build the fraction from the data

$$d = \frac{417 \text{ g}}{53.0 \text{ mL}} = 7.87 \text{ g/mL}$$

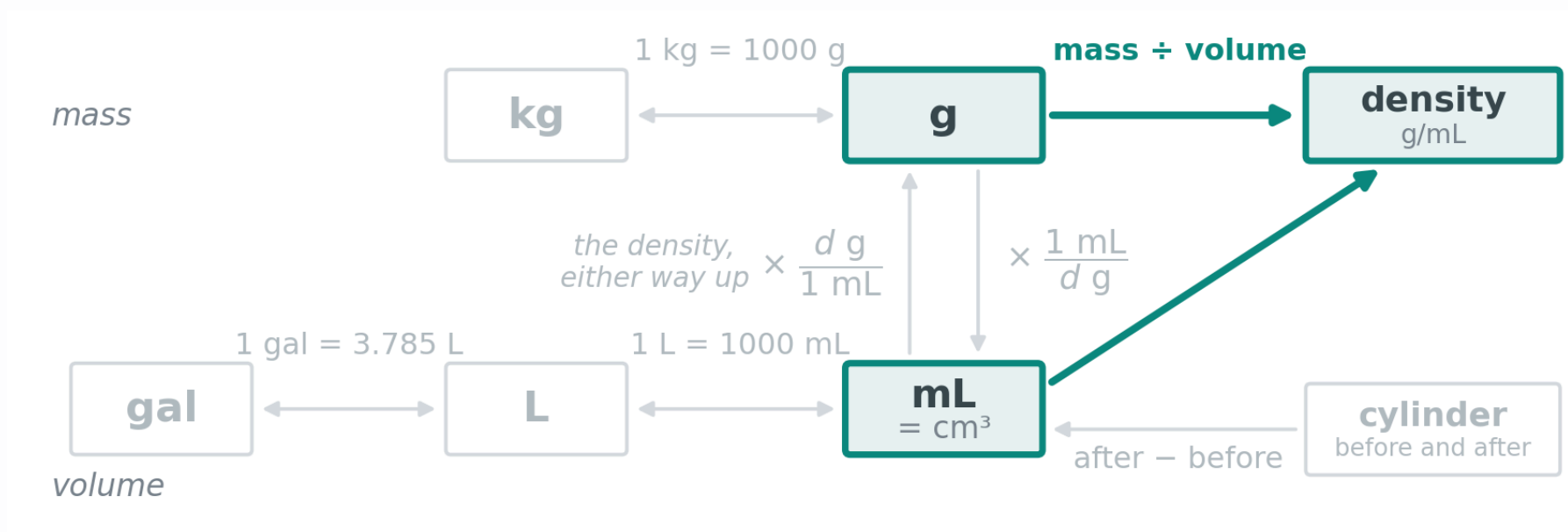
Match the property

aluminum	iron	copper	silver	lead
2.70	7.87	8.96	10.5	11.34

 **Sense check:** The measured fraction is now a conversion factor for these pellets: 7.87 g over 1 mL converts volume to mass; flipped, mass to volume. ✓

✓ Worked example 2: the route on the map

417 g of metal pellets · 53.0 mL of water displaced
found: 7.87 g/mL, iron



🧠 **Sense check:** No conversion this time. Mass ÷ volume builds the density from data in one move. ✓



Practice 3

109.3 g of metal · water level 21.4 mL → 33.6 mL

given: 109.3 g · two cylinder readings · wanted: g/mL

A **109.3 g** piece of scrap metal is lowered into a graduated cylinder of water. The water level rises from **21.4 mL** to **33.6 mL**. What is the density of the metal, in g/mL?

- A. 8.96
- B. 3.25
- C. 0.112
- D. 5.11

✓ Practice 3 · answer: A

109.3 g of metal · water level 21.4 mL → 33.6 mL

given: 109.3 g · 21.4 mL before · 33.6 mL after · wanted: g/mL

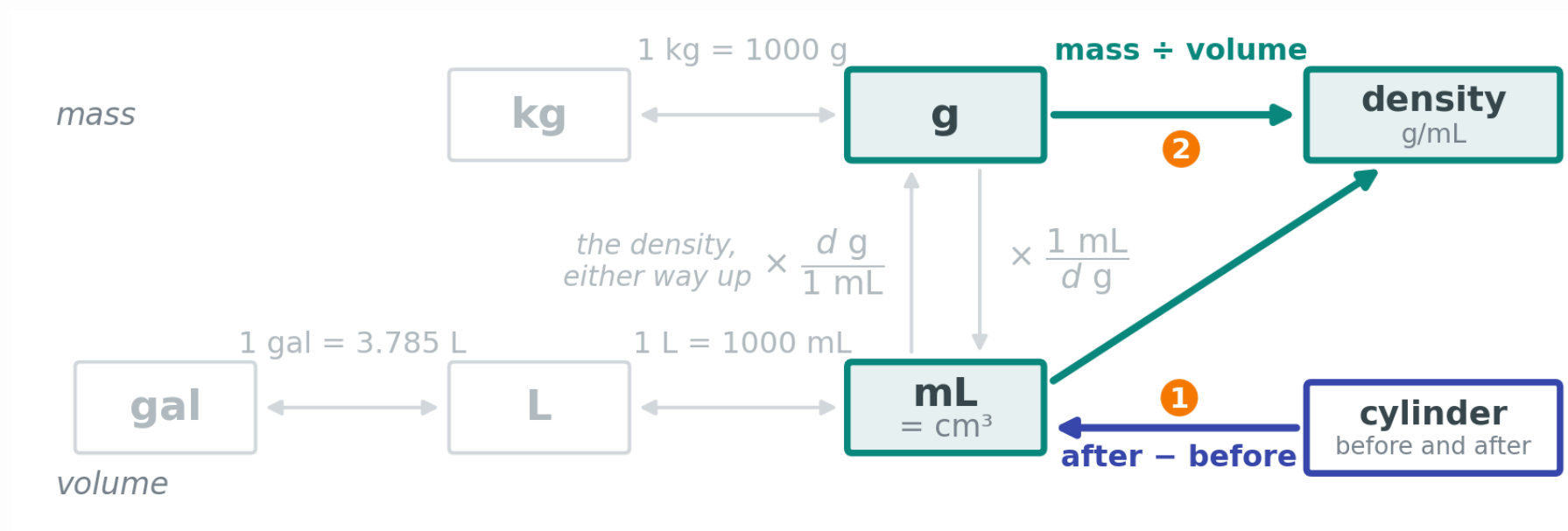
$$V = 33.6 \text{ mL} - 21.4 \text{ mL} = 12.2 \text{ mL} \rightarrow d = \frac{109.3 \text{ g}}{12.2 \text{ mL}} = \mathbf{8.96 \text{ g/mL (answer A)}}$$

B divided by the after reading: $109.3 \div 33.6 = 3.25$, but that volume counts the water too. C flipped the fraction: $12.2 \div 109.3 = 0.112 \text{ mL/g}$, not a density. D divided by the before reading: $109.3 \div 21.4 = 5.11$, the water alone.

🧠 **Sense check:** The metal fills only the 12.2 mL the water rose. 8.96 g/mL matches copper. ✓

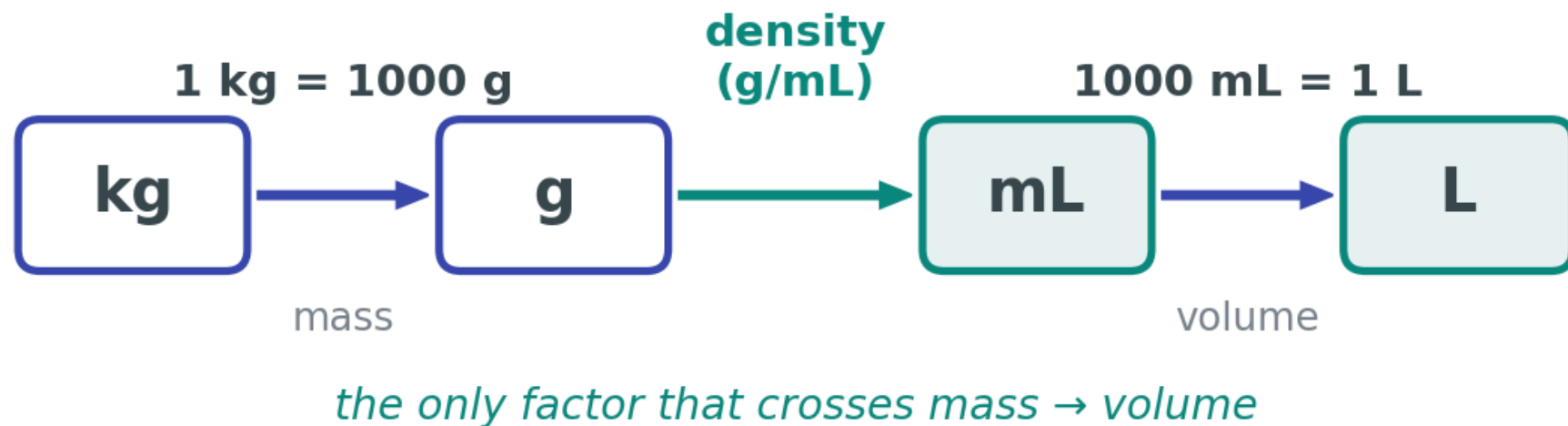
✓ Practice 3: the route on the map

109.3 g of metal · water level 21.4 mL → 33.6 mL
 given: 109.3 g · two readings · found: 8.96 g/mL



🧠 **Sense check:** Two moves: after – before gives the metal's volume, then mass ÷ volume. ✓

Density inside a longer route



Metric factors convert within mass or within volume. The density is the only factor that crosses between them. Map the route, then chain the factors on one rail.



Worked example 3: kilograms to liters

Step 1 • Map the route

$$0.789 \text{ g ethanol} = 1 \text{ mL ethanol}$$

given: 2.50 kg • wanted: L • route: kg $\rightarrow$ g $\rightarrow$ mL $\rightarrow$ L

A stockroom order arrives: **2.50 kg of ethanol**. The flammables cabinet is labeled in liters. How many liters is this?

The density carries only the g $\rightarrow$ mL arrow. Metric equalities carry the other two.

✓ Worked example 3: solution

0.789 g ethanol = 1 mL ethanol

given: 2.50 kg · wanted: L · route: kg → g → mL → L

Step 2 · Start with the given

Step 3 · Write the density fraction

Step 4 · Multiply and check

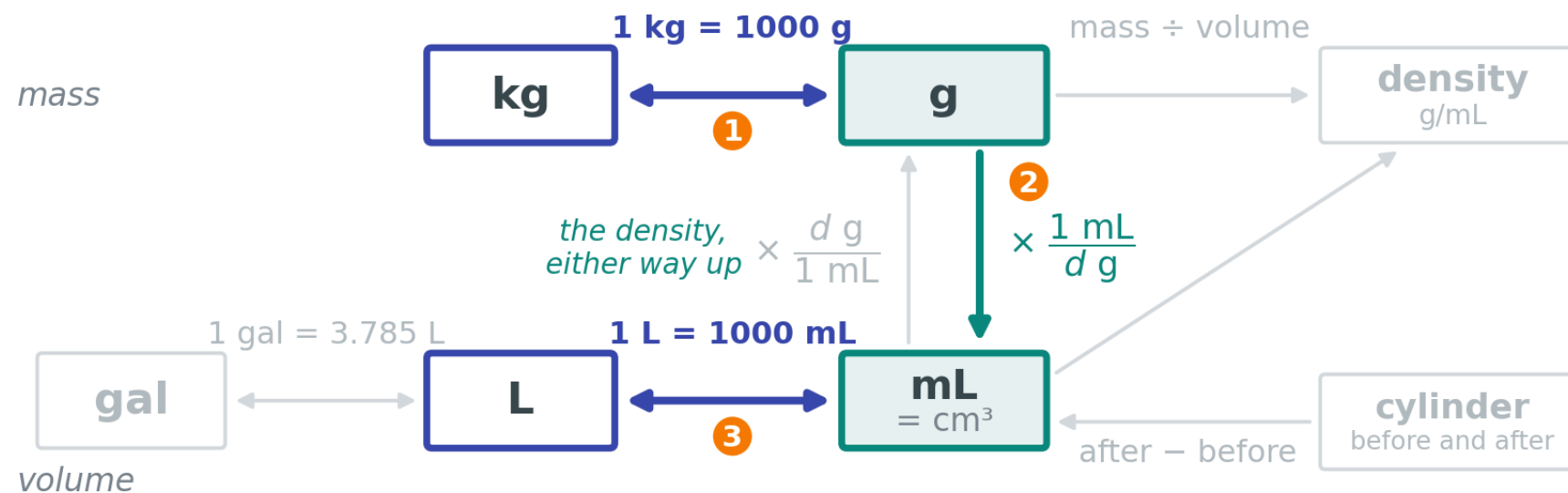
$$2.50 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} \times \frac{1 \text{ mL}}{0.789 \text{ g}} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 3.17 \text{ L}$$

 **Sense check:** Ethanol is lighter than water. Water would give exactly 2.50 L; ethanol spreads the same mass over 3.17 L. ✓

✓ Worked example 3: the route on the map

0.789 g ethanol = 1 mL ethanol

given: 2.50 kg · found: 3.17 L



🧠 **Sense check:** Three moves: kg → g, the density down to mL, then mL → L. Only the middle arrow crosses from mass to volume. ✓



Practice 4

$$1 \text{ gal} = 3.785 \text{ L} \cdot 1.84 \text{ g acid} = 1 \text{ mL acid}$$

given: 3.00 gal of concentrated sulfuric acid · wanted: kg

A stockroom receives **3.00 gal** of concentrated sulfuric acid, density **1.84 g/mL**. What mass, in kilograms, goes on the inventory sheet?

- A. 6.17
- B. 20,900
- C. 0.0209
- D. 20.9
- E. 1.46

✓ Practice 4 · answer: D

$$1 \text{ gal} = 3.785 \text{ L} \cdot 1.84 \text{ g acid} = 1 \text{ mL acid}$$

given: 3.00 gal · wanted: kg · route: gal → L → mL → g → kg

$$3.00 \text{ gal} \times \frac{3.785 \text{ L}}{1 \text{ gal}} \times \frac{1000 \text{ mL}}{1 \text{ L}} \times \frac{1.84 \text{ g}}{1 \text{ mL}} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 20.9 \text{ kg (answer D)}$$

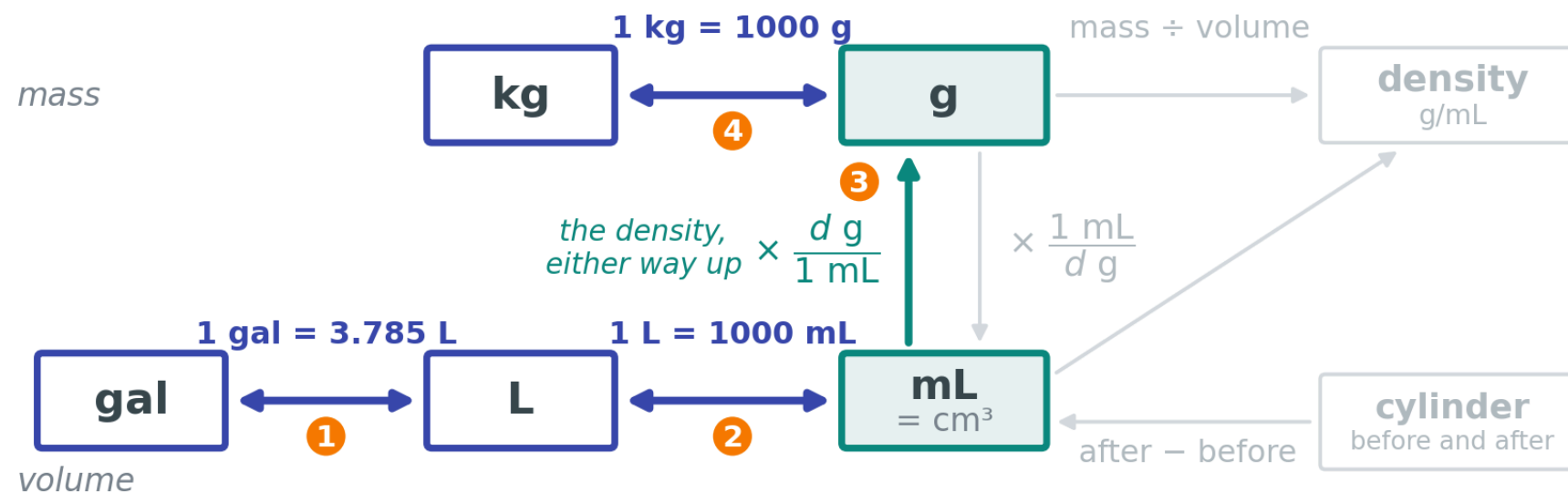
A flipped the density: $11,355 \div 1.84 \div 1000 = 6.17$. B stopped at grams: 20,893 g is the mass in g, not kg. C skipped L → mL, applying g/mL to 11.355 L: $20.9 \text{ g} = 0.0209 \text{ kg}$. E flipped the gallon factor: $3.00 \div 3.785 \times 1000 \times 1.84 \div 1000 = 1.46$.

🧠 **Sense check:** Water would be 11.4 kg; the acid packs 1.84 times the mass per milliliter: 20.9 kg. ✓

✓ Practice 4: the route on the map

$$1 \text{ gal} = 3.785 \text{ L} \cdot 1.84 \text{ g acid} = 1 \text{ mL acid}$$

given: 3.00 gal · found: 20.9 kg



🧠 **Sense check:** Four moves on one rail: gal → L → mL along the volume row, the density up to grams, then g → kg.

✓

 Practice 5

$2.70 \text{ g aluminum} = 1 \text{ cm}^3 \text{ aluminum} \cdot 0.740 \text{ g gasoline} = 1 \text{ mL gasoline}$
given: $0.250 \text{ L of aluminum}$ · wanted: mL of gasoline

What volume of gasoline, in mL, has the same mass as a **0.250 L** block of aluminum?

- A. 675
- B. 0.912
- C. 250.
- D. 68.5
- E. 912

✓ Practice 5 · answer: E

2.70 g aluminum = 1 cm³ aluminum · 0.740 g gasoline = 1 mL gasoline

given: 0.250 L aluminum · wanted: mL gasoline · route: L → cm³ → g → mL

$$0.250 \text{ L} \times \frac{1000 \text{ cm}^3}{1 \text{ L}} \times \frac{2.70 \text{ g}}{1 \text{ cm}^3} = 675 \text{ g} \rightarrow 675 \text{ g} \times \frac{1 \text{ mL gasoline}}{0.740 \text{ g}}$$

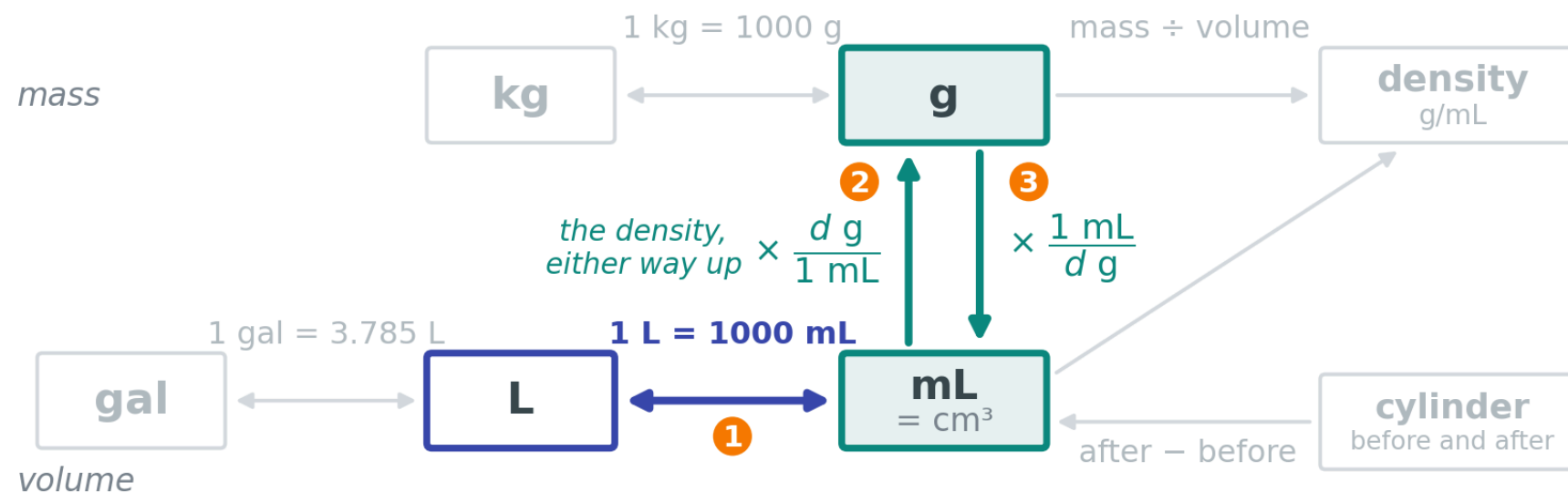
= 912 mL (answer E)

A stopped at the mass: 675 g. B left the volume in liters: $0.250 \times 2.70 \div 0.740 = 0.912$. C used aluminum's density for the gasoline too: 250. mL, but equal masses of different materials fill different volumes. D swapped the densities: $250. \times 0.740 \div 2.70 = 68.5$.

 **Sense check:** Aluminum is 3.65 times as dense as gasoline ($2.70 \div 0.740$), so the gasoline fills 3.65 times the 250. mL ✓

✓ Practice 5: the route on the map

2.70 g aluminum = 1 cm³ aluminum · 0.740 g gasoline = 1 mL gasoline
 given: 0.250 L aluminum · found: 912 mL gasoline



🧠 **Sense check:** Three moves: L → mL, up to grams with aluminum's density, back down to mL with gasoline's. Two materials, two densities. ✓

Check yourself

1. Iron: 7.87 g/mL. State the equality this declares, then write the fraction that converts **grams of iron to milliliters**.
2. A route runs **kg → g → mL → L**. Which arrow is the density's, and why can no metric factor replace it?

Molar mass is the next factor of this kind: the grams in one **mole** of a substance. One fraction on the same rail converts a mass into a count of particles.

Can you...?

- ☐ write and read numbers in scientific notation?
- ☐ report measurements and results with the correct significant figures?
- ☐ convert units with conversion factors, canceling units at each step?
- ☐ use density as a conversion factor between mass and volume?
- ☐ convert a temperature among $^{\circ}\text{C}$, $^{\circ}\text{F}$, and K, and handle a temperature difference?

If any box stays empty, the practice site has a drill for it. 